An analogy between a connected exceptive phrase and polarity items

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1. Introduction
Exceptive phrases (EPs), as in (1), serve fundamentally to remove objects from the domains of quantifiers. EPs come in a variety of forms. They are headed by different exceptive markers (EMs), such as but, except, besides. And, as Hoeksema (1987) first observed, they divide into two syntactic classes. Connected exceptives have the distribution of post-nominal modifiers, whereas free exceptives have the distribution of sentential adverbials. While domain subtraction is the core of the semantics of EPs, they may carry additional components to their meaning, including what might be described as entailments of exception. (1a) entails that Bill doesn’t smoke; (2a) that Bill does. Furthermore, even though domain subtraction could in principle be performed on any quantifier, many exceptive quantifiers are quite picky about which quantifiers’ domains they will modify. So, while (1a) and (2a) are acceptable, the sentences in (2b) are not.

(1) a. Every student but Bill smokes.
b. No student smokes, except John.
c. Besides John, two students smoke.

(2) a. No student but John smokes.
b. *Two/*many/*most students but John smoke.

(3) **Exceptive Terminology**

\[
\begin{array}{c}
\text{Exceptive Marker (EM)} \\
\text{Every student but Fred answered the question correctly.} \\
\text{Exceptive Phrase (EP)}
\end{array}
\]

In this paper, I will focus specifically on the English exceptive marker but and the distribution of the connected EPs that it heads. My goal is to offer a deeper explanation of the distribution of EPs headed by but than those currently available. This will lead me to develop an analogy in the analysis of EPs headed by but, henceforth but-EPs, with recent developments in the analysis of negative polarity and free choice items. That is, I will suggest that but-EPs’ restricted distribution derives from the same mechanisms that restrict the distribution of such sensitive indefinites. Specifically, I will argue that but introduces a highly structured set of alternatives, which are then semantically exploited recursively by a covert exhaustivity operator. The presence of the covert operator is guaranteed by the featural requirements of the exceptive marker.

But-EPs, are quite limited in distribution. It is generally assumed that they can only associate with universal quantifiers (like every) and universal negative quantifiers (like no), cf. Hoeksema (1987), von Fintel (1993), Moltmann (1995). Recent work on exceptives by Peters & Westerståhl (2006) and García-Álvarez (2009) has questioned the restricted distribution of exceptives. The vast majority of the examples produced to argue
against a restriction to universal (negative) quantifiers involve the exceptive marker except (for). As far as I can tell, but’s distribution is in fact limited to universal (negative) quantifiers, as illustrated in (4). There may be dialectal and lexical differences at work in the variations in judgments.

(4) All/No/*Most/*Many/*Some/*Few/*At most 3 boys but Phil smoke.

As hinted at above, the interpretation of a but-EP implies that the individuals named by its complement belong to the restrictor of the associate but take a different value from the other members of the restrictor with respect to the predicate that is the scope of the quantifier.

(5) a. Every student but Mary smokes
   b. No student but Mary smokes.

(6) a. Both (5a) and (5b) imply that Mary is a student.
   b. (5a) implies Mary does not smoke
   c. (5b) implies Mary does smoke

One important goal in the study of exceptives has been to try to connect the interpretation of EPs to their distribution, see especially von Fintel (1993) and Moltmann (1995). I will build my analysis primarily on the idea of von Fintel (1993) that the meaning of exceptives has two parts: (i) set substraction and (ii) a uniqueness condition on the set subtracted from the quantifiers domain. In this paper, I follow up on a suggestion in Gajewski (2008) that the uniqueness condition is enforced by an exhaustivity operator that can take scope independent of the associate quantifier of the but-EP. In this, I take inspiration from work on distributionally restricted indefinites like negative polarity and free choice items by Chierchia (2004) and Kratzer and Shimoyama (2002) and many subsequent scholars. To be more specific, von Fintel argues that but implies that its complement denotes the unique minimal set of individuals that can be subtracted from the domain of a quantifier to make the resulting quantificational statement true. I will argue that the minimality of the complement set and the uniqueness of that minimality are each enforced by an exhaustivity operator. I draw direct inspiration from Chierchia’s work on the distribution of any. Chierchia argues that any introduces alternatives to its contextual restriction and that these are subject to exhaustification. Furthermore, Chierchia argues that the alternatives to the contextual restriction are subsets of the restriction (see also Krifka 1995). This structuring of the set of alternatives will play an important role in my analysis of but.

Interestingly, on the surface, the distribution of but-EPs resembles that of polarity items: they occur in the restrictors of universal (negative) determiners and of any. When any is free choice it is arguably universal. The restrictors of universals are anti-additive (AA).

(7) No/Every/Any student but Sue could solve this problem.

(8) A function F of type <σ,τ> where σ and τ are Boolean types is anti-additive iff
For any $A$, $B$ of type $\sigma$: $F(A \lor B) \leftrightarrow F(A) \land F(B)$

When a but-EP attaches to NPI $any$, it is in an NPI licensing environment, of course, and furthermore this combination is most acceptable in the scope of an AA operator, cf. (9). See Hoeksema (1987, 1995) for the suggestion of a connection between connected EPs and AA-ity.

(9) *At most 3/4 Few/No students ate anything but halibut.

At least superficially this resembles the distribution of some strong NPIs, cf. Zwarts 1998. Many strong NPIs, however, are uncomfortable in AA restrictors (but compare (11)):

(10) Every student Bill taught passed.
    *Every student SUE taught either passed.
    ?Every student who has taken it in years passed the exam.

(11) Every student with so much as a dime has a smart phone.

The connection I’ll suggest between EPs and NPIs is much less direct than saying that EPs are strong NPIs. In a nutshell, here again is the proposal that I will be pursuing:

Proposal
1. The EM but denotes set subtraction.
2. but-EPs trigger the introduction of alternatives

Since the proposal relies heavily on von Fintel’s (1993) analysis of exceptives, we turn to an examination of his proposed semantics in the next section. Once we have reviewed von Fintel’s account and its problems, the new proposal is laid out in section 3. Residual issues raised by the proposal are addressed in section 4. Section 5 concludes.


The account of the meaning of connected exceptives given in von Fintel (1993) achieved two major goals. First, it gave the first unified account of positive and negative universal cases. Second, it sought to derive the distribution of connected exceptives from the proposed truth conditions. To understand von Fintel’s account it will be useful to introduce some terminology concerning exceptive constructions:

(12) Some useful terminology:
1. A quantification: a determiner together with its restrictor and scope. $D(A)(P)$
2. An exception set to a quantification in a world $w$: $C$ is an exception to a quantification in a world $w$, if $D(A-C)(P)$ is true in $w$. 
The essential semantic function of an EM is to subtract its complement from the restrictor. This alone, however, yields a semantics that is too weak. The statement in (13a) entails that Bill smokes, not merely that every student who isn’t Bill smokes. Thus, von Fintel argues that the subtraction performed by *but*-EPs is subject to further conditions.

(13) a. Every student but Bill smokes.
    b. Every student smokes.

In particular he argues that it is encoded in *but*’s semantics that its complement denotes the unique minimal exception set to the quantification. An exception set is minimal if it has no proper subsets that are exception sets. A minimal exception set is unique if it is the only minimal exception set. In the case of (13a), if we know that \{Bill\} is a minimal exception set, then we know that (13b) is false – since this is equivalent to subtracting the empty set, a proper subset of \{Bill\}, from the set of students. If (13a) is true and (13b) is false, then we can infer that Bill is a student and does not smoke.

Uniqueness comes into play primarily in ruling out cases where an EP modifies an inappropriate associate quantifier. Let’s see how minimality and uniqueness work together to rule out associates that are not universal (negative) quantifiers. First, minimality: Left upward entailing (LUE) determiners have only trivial, empty minimal exception sets. Suppose D is LUE and C is a non-empty exception set to D(A)(P), i.e., D(A-C)(P) is true. Since Ø⊆C and A-C ⊆ A-Ø, D(A-Ø)(P) is true. Therefore, C is not a minimal exception set. Von Fintel argues that sentences like (14) that are necessarily false are unacceptable, see Gajewski (2002) for development of this point. Note the unacceptability of (14) with LUE determiner *some*.

(14) *Some student but Fred smokes.

Quantifiers other than *every* and *no* may have minimal exception sets. For example, quantifiers headed by *at most three* and *exactly three* can have minimal exception sets in certain contexts. Consider a scenario in which exactly four students smoke. In such a scenario, a set consisting of exactly one of the smoking students would be a minimal exception set for (15b) and (15c). It is easy to see why. If we remove one smoking student from the set of students, when there are exactly four smoking students, then the cardinality of the intersection of the set of smokers with the set of students – minus one smoker! – will be three. Thus, the conditions of the determiners *at most three* and *exactly three* are satisfied. Furthermore, that singleton set containing a student smoker has no subsets that we could subtract from the restrictor and get a true statement. After all the only proper subset of such a singleton set is the empty set, and if we don’t subtract anything from the set of students, we will get a false statement. There are four student smokers and that is not compatible with the truth conditions associated with *at most three* and *exactly four*. So, in certain contexts, quantifiers headed by *at most three* and *exactly four* can have minimal exception sets.

(15) a. Every student smokes.
b. Exactly three students smoke.
c. At most 3 students smoke.

Notice, however, that a unique minimal exception set cannot be identified in this case. If there are four student smokers, any of them could serve as the single exception we need to get to a true statement. Compare this with the case of every. When the statement in (15a) is false, the unique minimal exception set is exactly the set of students that don’t smoke. When the statement in (15b) is false, however, any set consisting of n smoking students is a minimal exception when n+3 is the number of smoking students. This is the reason that von Fintel builds a uniqueness condition on the minimal exception set into this semantics. Among the quantifiers that permit minimal exception sets, very few give rise to unique minimal exception sets.

All these considerations led to von Fintel’s analysis of the complement of but as the unique minimal, or least, exception set. I am assuming that the EP is the syntactic sister of the restrictor of the quantifier. In the lexical entry below, C is the set contributed by the complement of but, A is the denotation of the restrictor of D, and P the scope.

\[(16) \text{ Von Fintel’s lexical entry:} \]

For any C, A, P of type \(\langle e,t \rangle\) and D of type \(\langle et,<et,t> \rangle\):

\[\langle \text{but} \rangle(C)(A)(D)(P) = 1 \iff \exists S[D(A-C)(P)=1 \& \forall S[D(A-S)(P)=1 \rightarrow C \subseteq S] \]

\[\text{Subtraction} \quad \text{Unique minimality} \]

\[\]

2.1. Problems for von Fintel (1993)

Despite its successes, there are a few problems for von Fintel’s account as it is stated. Mainly, there are reasons to doubt that unique minimality is built into the lexical entry of but itself. First, consider the fact that a but-EP can combine with an associate quantifier headed by NPI any.

\[(17) \text{ Observation 1: but-EPs can combine with NPI any.} \]

\[(18) \text{ Nobody ate anything but halibut.} \quad (\text{Gajewski 2008}) \]

There are many good arguments in the literature that NPI any is an existential quantifier Ladusaw (1979), Carlson (1980), Linebarger (1981). Furthermore, but-EPs are acceptable with NPI any in those environments that diagnose a weak, existential semantics for that determiner. Consider the occurrence of a but-EP modifying NPI any in the sentence below.

\[(19) \text{ I don’t think there’s anything but beer in the fridge.} \]

This is problematic, since as we observed above von Fintel’s semantics predicts incompatibility between connected EPs with LUE determiners.

Next observe that von Fintel’s (1993) semantics predicts that the constituent \([\text{no NP but DP}]\) should denote a nonmonotone generalized quantifier. There is no space to
go through the derivation in detail, but von Fintel’s analysis specifically predicts that such a quantifier is not monotonic in its scope. If (20) maps a set to 1, that set must contain Bill. Every such set however will have a subset that does not contain Bill; (20) maps all such sets to 0. So, (20) is not downward entailing (DE). Furthermore if (20) maps a set to 1, it must contain no students other than Bill. However, in any situation in which there are students other than Bill, (20) will map all such sets to 0. So, (20) is not upward entailing (UE). Thus, (20) is nonmonotone.

(20) \([\text{no student but Bill}] = \lambda P_{<e,\ell}. \{x: x \text{ is a student and } P(x) = 1\} = \{\text{Bill}\}\]

This furthermore predicts that such a constituent should be a poor licensor of NPIs, no better than other nonmonotone quantifiers like \([\text{exactly } n \text{ NP}]\). \([\text{exactly } n \text{ NP}]\) only licenses NPIs in very specific contexts, cf. Linebarger (1980), Crnič (2011). This prediction is not borne out. \([\text{no NP but DP}]\) is in fact quite a good licensor of NPIs. Consider, for example, that \([\text{no student but DP}]\) is quite similar in meaning. As is well known, \([\text{only DP}]\) licenses weak NPIs such as \(\text{any}\) and \(\text{ever}\). So, one might think that theories that explain \([\text{only DP}]\)’s licensing abilities may also explain \([\text{nobody but DP}]\)’s licensing abilities. This cannot be however since these two phrases show different licensing properties. \([\text{nobody but DP}]\) licenses strong NPIs, like \(\text{either}\), but \([\text{only DP}]\) does not, (23).

(21) a. Exactly three students said anything in my class.
   b. Nobody but Susan said anything in my class.

(22) Observation 2: Though \textbf{nobody but Al} is roughly equivalent in meaning to \textbf{only Al}, they differ in their ability to license strong NPIs. (Gajewski 2008)

(23) a. Only Bill likes pancakes. *Only Bill likes WAFFLES, either.
   b. Nobody but Al likes pancakes. Nobody but Al likes WAFFLES either.
   (Nathan 1999)

The fact that \textit{but}-EPs combine felicitously with free choice \textit{any} may also pose a challenge for von Fintel’s account. Recent accounts posit that free choice \textit{any} is an existential quantifier whose universal flavor derives from a wide scope pragmatic operator, cf. Chierchia (2006), Fox (2007), Menendez-Benito (2010). If such views are correct, then this is also a case of a \textit{but}-EP combining with an LUE determiner.

(24) Any student but John can pass this exam.

(25) \textit{but} EPs combine with free choice \textit{any}. Recent theories posit that free choice \textit{any} is underlyingly indefinite; universality derives from a wider scope pragmatic operation.

Gajewski (2008) argues that these observations point to the unique minimality condition on \textit{but}-EPs being imposed at a scope site higher than the scope of the \textit{but}-EP’s associate quantifier. This can explain \textit{but}-EPs’ co-occurrence with NPI \textit{any}, since the
unique minimality can be enforced above the negative licenser. Furthermore, it explains why [no NP but DP] has the same licensing properties as [no NP], since the monotonicity-destroying unique minimality condition is imposed at a higher scope level than the NPI licenser. Likewise, in the free choice case, the unique minimality should be imposed above the pragmatic operator, at the point where universality is achieved.

Gajewski (2008) offers an elaboration of von Fintel’s semantics that allows for unique minimality to be imposed separately from subtraction. Gajewski’s semantics separates the set subtraction and unique minimality components of but’s meaning into two separate operators. The first, but itself, simply encodes set subtraction. The second is a covert operator called LEAST (= unique minimality). Gajewski formulates a rule that requires but to be locally c-commanded by LEAST. LEAST thus takes wider scope than the exceptive marker itself. For simplicity, I give the version of Gajewski’s theory in which LEAST originates in the complement of but and takes syntactic scope due to a type mismatch. Gajewski (2008) ultimately adopts a different theory in which LEAST is a focus sensitive operator.

\[
\begin{align*}
(26) & \quad a. \mathbb{[but]} = \lambda X_{et} \lambda Y_{et}. Y - X \\
& \quad b. \mathbb{[LEAST]} = \lambda X_{et} \lambda F_{.et,t}. F(X) = 1 & \forall S [F(S) = 1 \rightarrow X \subseteq S]
\end{align*}
\]

\[
(27) \quad \text{No student but Bill smokes}
\]

LF:

\[
\begin{array}{c}
\text{S} \\
\vdash \text{[LEAST Bill]}_{1, <e,t>} \\
\text{S} \\
\text{DP} \\
\text{VP} \\
\text{D} \\
\text{NP} \\
\text{no} \\
\text{N} \\
\text{PP} \\
\text{smokes} \\
\text{P} \\
\text{t}_{1, <e,t>} \\
\text{but}
\end{array}
\]

I leave it to the reader to see that for any determiner Det, noun phrase NP, verb phrase VP and set-denoting expression C, the structure in (28a) with von Fintel’s meaning for but is truth conditionally equivalent to (28b) with Gajewski’s meaning for but. The structure of course is different and this is the key to solving the problems noted above. The next section shows how to resolve these problems in Gajewski’s theory.

\[
(28) \quad a. \mathbb{[[ Det \ NP but_{et,F} C ] \ VP]} \\
& \quad b. \mathbb{[LEAST C]_{X} [[ Det \ NP but_{G} X ] \ VP]}
\]

2.2. Solutions to von Fintel’s problems

Below, I illustrate more clearly the solutions to the three problems for von Fintel’s account. In (29), we see that LEAST can take scope above a NPI licenser. In this way, it
is as if the *but*-EP were associated with a quantifier headed by *no*, although it is syntactically composed of *not* and *any*. The point is that after movement and abstraction the interpretation of the sister of [less *Al*] is (30). This is equivalent to the interpretation the sister of [less *Al*] in (31) where the EP is contained in a DP headed by *no*.

(29) Bill didn’t see any student but Al.
    LF: [less *Al]*x [not [Bill saw any student but x ]]

(30) λx<e,t>. Bill did not see a student who is not in X

(31) Bill saw no student but Al.
    LF: [less *Al]*x [Bill saw no student but x ]

This allows us to assign the correct interpretation to sentence in which an EP is contained in a quantifier headed by NPI *any*. See Gajewski (2008) for constraints that prevent EPs from occurring with non-NPI indefinites in the scope of negation.

Now we turn to the case of NPI licensing by DPs headed by *no* that contain an EP. Under Gajewski’s account, the quantifier containing the EP on the surface no longer forms a constituent with all semantic components of the EP. For example, in (32), we see that least, which enforces unique minimality, takes scope at LF outside the phrase headed by *no*. The constituent left behind after movement of [least *Al*], namely [no body but X], denotes an anti-additive generalized quantifier, for any value assigned to the variable X. Following Zwarts 1998, we assume that anti-additivity is sufficient to license strong NPIs like *either*. Thus, we account for the licensing properties of [no NP but C], for any noun phrase NP and set denoting expression C.

(32) Nobody but Al likes WAFFLES, either.
    LF: [least *Al]*x [Nobody but X like waffles either]

Finally, we see in (33) that a *but*-EP can behave semantically as if it were modifying a universal quantifier, so long as the least portion of its meaning can take scope above the operator that gives a free choice indefinite its universal flavor, which I designate generically as OpFC. I leave this point as a hint to the reader. This is not the place to get into the intricacies of current debates on free choice indefinites (although Section 3.2.3 below will offer some further hints). I refer the reader to works that suggest that free choice interpretation is established by operators taking wider scope than the free choice indefinites themselves: Chierchia (2006), Fox (2007), Menendez-Benito (2010). Also see Gajewski (2008) for more detailed discussion of the scope properties of the covert operator least.

(33) Bill may have any flavor of ice cream but coffee.
    LF: [least coffee]*x OpFC [may [ Bill have any flavor but x ]]

8
2.3. Question for Gajewski (2008)

The idea that the unique minimality part of the meaning of a but-EP takes scope independently of the set subtraction part provides an interesting solution to several problems. However, the idea also raises some questions. In particular, one must ask why the meaning of the exceptive phrase is split in two. The idea pursued in Gajewski (2008) is that the fundamental part of an exceptive’s meaning is set subtraction and that the unique minimality part is similar in nature to many pragmatic inferences of uniqueness or exhaustivity. It is difficult to see, however, how LEAST relates to any known exhaustivity operator. Consider the standard definition of an exhaustivity operator below (see e.g. Chierchia 2006):

\[ ([\text{EXH}](C)(p) = \lambda w. p(w)=1 \& \forall q[ q \in C \& q(w)=1 \rightarrow p=q ] \]

Here the exhaustivity operator applies to a proposition against a background of alternative propositions – the operator can of course be defined recursively to apply to other Boolean types. The point of the exhaustifier is to exclude alternative propositions. In the simple formulation in (34), all alternatives not entailed by the argument proposition, also known as the prejacent, are denied. The set of alternative propositions may be provided, for example, by focus or by a Horn scale (see Horn 1968 et seq, Sauerland 2004 for recent discussion). Recent work has also raised the possibility that lexical items might introduce sets of alternatives as a part of their conventional lexical meanings.

The operator LEAST, defined in (26b), is clearly related to operators like (34), but is also crucially different. While not strictly speaking defined as operating on proposition, it is not difficult to see how it could be viewed as excluding alternative proposition. LEAST takes an argument X of type \(<e,t>\) and an argument F of type \(<<e,t>,t>\). The truth conditions imposed by LEAST include the requirement that F applied to X maps to 1, and that F applied S yields 0 for any set S that is not a superset of X. These alternative ‘propositions’ (think F(S) for any set S) to be excluded are not selected by their logical relation to the ‘prejacent’ (think F(X)), rather it is dictated that they are propositions in which the exception set has been replaced by a subset. My goal in this paper is to assess the prospects for replacing LEAST with EXH, and thereby assimilating but-EPs to other expressions that apparently require the presence of EXH. In particular, this will situate the explanation of the distribution of but-EPs within the emerging paradigm of explaining the distribution of polarity items spearheaded by Chierchia (2006, 2010). The hope is that by assimilating but-EPs to other expressions whose semantics is dependent on EXH we will have a deeper understanding of the limitations on but-EPs distribution.

3. A new approach to exhaustification and exceptives

In this section, we undertake the task of providing a deeper explanation for the distribution of certain exceptive phrases. I begin in Section 3.1 by considering and rejecting the obvious idea of simply substituting EXH for LEAST in the semantics of but-
EPs. I then develop a new semantics for but-EPs based heavily on von Fintel’s original conceptual description of the semantics of EPs (Section 3.2) and on Fox’s recent theory of free choice disjunction that makes use of the novel concept of recursive exhaustification (Section 3.3). Then in Section 4, I discuss some conditions on the possible scope of the exhaustification associated with but-EPs.

3.1. A first attempt

The first idea that comes to mind is simply to replace LEAST with EXH. Under such a theory, but itself denotes set subtraction, introduces alternatives to the exception set contributed by its complement and carries a feature requiring the local presence of an EXH to exploit that set of alternatives. This simple idea will not work, however. Consider the structure below. A quick note on notation as we proceed. In the tree below and what follows I will use the italicized category labels that appear as terminal nodes as metalinguistic variables that range over expressions that may occur in these positions. In particular ‘Det’ ranges over quantification determiners of type <<e,t>,<<e,t>,t>>, ‘NP’ ranges over any nominal expression of type <e,t>, ‘VP’ is meant to range over any main predicate that has a denotation of type <e,t>, and ‘C’ ranges over expressions of type e – I follow von Fintel (1993) in assuming that these type e expressions are type-shifted to denote sets that can serve as appropriate arguments for but, which denotes set subtraction. I furthermore make the simplifying assumption that C is a rigid designator. Thus, below I will omit the world subscript from $[C]$, as I do from the logical $[Det]$.

$$(35)\quad \begin{array}{c}
\text{EXH} \\
S \\
\text{Det} \\
NP \\
\text{but}_{[+\text{EXH}]} \\
C
\end{array}
$$

$$(36)\quad [\text{but}] = \lambda X_e. \lambda Y_e. X \subseteq Y, Y \setminus X
$$

When the determiner Det in the configuration in (35) is every or no, the correct result obtains. The reason is that both determiners are Left Downward Entailing (LDE). For example, an alternative proposition like (37b) is entailed by the prejacent (37a) if and only if $[C] \subseteq S$, for any set of individuals S. If a set S is a superset of $[C]$, then in any world w, $[NP]^w \setminus S$ is a subset of $[NP]^w \setminus [C]$. That is, (37a) entails (37b). If S is not a superset of $[C]$, then there is some world u such that $[NP]^u \setminus [C] \subseteq [VP]^u$, but $[NP]^u \setminus S \not\subseteq [VP]^u$. That is, (37a) does not entail (37b).

$$(37)\quad \begin{array}{l}
a. \lambda w. [\text{every}]( [NP]^w \setminus [C]) ([VP]^w) \\
b. \lambda w. [\text{every}]( [NP]^w \setminus S) ([VP]^w)
\end{array}$$
Hence, in these cases, the use of entailment to select which alternatives are excluded reduces to von Fintel’s use of the subset relation among alternative exception sets. But this successful reduction holds only in the case of LDE determiners. Consider the case of an LUE determiner, such as *some*. Recall that *but*-EPs are unacceptable with LUE determiners, except for *any*.

\[(38) \quad *\text{Some student but Bill smokes.}\]

\[(39) \quad \begin{align*}
    a. & \quad \lambda w. [\text{some}][\text{NP}]w - [\text{C}]( [\text{VP}]w) \\
    b. & \quad \lambda w. [\text{some}][\text{NP}]w - S)( [\text{VP}]w)
\end{align*}\]

In this case, an alternative proposition like (39b) is entailed by the prejacent (39a) if and only if \(S \subseteq [\text{C}]\), for any set of individuals \(S\). This then reverses von Fintel’s subset condition. So, any alternative that involves an alternative exception set that is not a subset of \(C\) is excluded. Given this, (40) is the denotation of (35) substituting *some* for *Det*. The question now is whether this guarantees a contradiction, explaining the unacceptability of (38) in the manner of von Fintel (1993).

\[(40) \quad \lambda w. [\text{some}][\text{NP}]w - [\text{C}]( [\text{VP}]w) = 1 & \forall S[[\text{some}][\text{NP}]w - S)( [\text{VP}]w) = 1 \rightarrow S \subseteq [\text{C}]\]

Somewhat surprisingly, however, this is not a contradiction. This can be shown by constructing a model that satisfies this proposition. Consider the model of a world \(u\) below. The domain of this model \(D_u\) contains only two individuals, a and b. (40) is true in \(u\), i.e. maps the world \(u\) to 1. There are two sets of individuals in the model that are not subsets of \([\text{C}](=\{b\})\). Those are \{a\} and \{a,b\}, cf. (42). However, subtracting either of those sets from \([\text{NP}]u\) leaves a set that has no overlap with \([\text{VP}]u\). So, every exception set \(S\) that is not a subset of \([\text{C}](=\{b\})\) makes the prejacent statement false, i.e., makes it so that \([\text{some}][\text{NP}]w - S)( [\text{VP}]w) = 0\). Hence, as required by the semantics of \(\text{EXH}\), all alternative propositions that are not entailed are in fact false in \(u\).

\[(41) \quad D_u = \{a, b\} \]
\[ [\text{NP}]u = \{a, b\} \]
\[ [\text{VP}]u = \{a\} \]
\[ [\text{C}] = \{b\} \]

\[(42) \quad \{S \subseteq D_u : S \not\subseteq \{b\}\} = \{\{a\}, \{a,b\}\} \]

One response to (41) would be to attack this model and suggest that it is not a possible domain. The task is complicated by having to rule out such models for all non-LDE determiners and not just for *some*. I have not uncovered any principles that would rule out this diversity of models. Consequently, I will pursue another route to using exhaustification to implement the semantics of *but*-EPs.

3.2. Reconsidering von Fintel’s semantics
In this section, I return to von Fintel’s original description of \textit{but}-EP’s semantics in terms of unique minimality. As described in the introduction, minimality of the exception set plays a crucial role in the semantics of \textit{but}-EPs, but is not sufficient to derive their distribution. If \textit{but} merely subtracted its complement from a quantificational restrictor, then (43a) would be predicted, incorrectly, not to entail that Fred does not smoke. Furthermore, LUE determiners have no non-empty minimal exception sets. So, minimality can already exclude all LUE determiners as \textit{but}-EPs associates.

(43) \hspace{1em} a. Every student but Fred smokes
    b.*Some student but Fred smokes

Minimality does not suffice, however, to rule out non-AA LDE determiners, such as \textit{at most three}, or non-monotone determiners, such as \textit{exactly three}. Each of the determiners below can have minimal exception sets. This was demonstrated in some detail above in Section 2 concerning examples (15b,c). I simply remind the reader here that in a scenario in which exactly four students smoke, any set containing exactly one of the smoking students would be a minimal exception set for the statements in (44).

(44) \hspace{1em} a. *At most three students but Fred smoke.
    b. *Exactly three students but Fred smoke.

Notice though that in this case there are four minimal exception sets. Von Fintel’s (1993) idea is that this is to be excluded: \textit{but}-EPs can only be used felicitously when there is a \textit{unique} minimal exception. The idea that I pursue here is to implement this idea in the following way: the minimality and the uniqueness in the semantics of \textit{but}-EPs are each instances of exhaustification. Thus, I will propose that \textit{but}-EPs are exhaustified recursively. In the next two sections, I show how this can be accomplished. Section 3.2.1 analyzes minimality in terms of exhaustification. Section 3.2.2 analyzes uniqueness in terms of exhaustification.

3.2.1. Minimality

First, I formalize minimality of the exception set as a form of exhaustification. As before, I take the basic meaning of \textit{but} to be set subtraction, (45). The next step is to formalize the set of alternatives that are introduced by the \textit{but}-EP. Here I follow Krifka (1993) and Chierchia (2004) \textit{et seq} in restricting the set of alternatives to a domain as the set of subsets of that domain (46). This is a way to still use exhaustification, (47), but to get around the problem of reversal of direction observed in Section 3.1, namely that the selection of alternatives to be excluded by determining which alternatives are entailed yields incorrect results in the case of non-LDE determiners. How we get around this problem will be demonstrated below. First, the denotation of the exceptive marker remains the same as above in (36) – set subtraction, cf. (45). Next, we stipulate that the alternatives to an exceptive phrase are denotations for the exceptive phrase in which the extension of the complement of the exceptive marker is replaced by a subset, cf. (46). Finally, we adopt a standard meaning for the exhaustive operator \textit{EXH}, (47).
Let us see how this works in the simple case of the LDE determiner. The alternatives introduced by the but-EP project as propositional alternatives to the propositional complement to EXH and serve as the antecedent for the operator’s contextually determined set-of-propositions argument. It is this set that provides the propositions that are to be (potentially) excluded by the exhaustification. Now, I have proposed that we limit the alternatives to cases of propositions involving exception sets smaller than that of the prejacent. Of course, the smaller the exception set, the larger the restriction. In the case of an LDE determiner, this means that none of the alternatives are entailed, save those that are equivalent to the prejacent. For example, the second alternative listed in (48b) – distinct from the prejacent – is not entailed and, thus, is excluded.

No student but Bill smokes
a. EXH(A) (λw.[no](some)¬[Bill])([smokes])
b. A = { λw.[no](some)¬[Bill])([smokes]), λw.[no](some)¬Ø)([smokes])}
c. No student who is not Bill smokes and some student smokes.

It follows, as desired, from the exclusion of this alternative that Bill is a student who does smoke. Let us now turn to the case of a LUE determiner like some. In this case, matters are a little more complicated. As the reader will recall from Section 3.1, if all alternative exception sets are considered, the alternative proposition excluded are those that involve an exception set that is not a subset of the prejacent’s exception set. Now, however, we are limiting the range of alternatives exclusively to those that involve subsets of the prejacent’s exception set.

*Some student but Bill smokes.
a. EXH(A) (λw.[some](student)¬[Bill])([smokes])
b. A = { λw.[some](student)¬[Bill])([smokes]), λw.[some](student)¬Ø)([smokes])}

So, in the case of some all available alternative propositions are entailed. I would like to propose that there is a condition (a presupposition, perhaps) on the use of but-EPs that is violated when none of the alternatives introduced are eliminated. This condition may be related to a condition that Chierchia (2010) has recently proposed as a restriction on some polarity items, Anti-Idleness. Alternatively, this principle may be considered related to principles of economy: the presence of but triggers the presence of an exhaustifier, but the exhaustifier fails to eliminate any alternatives.
At this point, as predicted, we obtain the correct meaning for cases where the associate quantifier is headed by *every* or *no* and we correctly rule out associate quantifiers headed by LUE determiners like *some*. We are not finished, however. We derive a consistent meaning for (50), see (50c). This is problematic since the sentence is unacceptable, having an associate of a *but*-EP headed by a left non-monotone determiner. For readability, I introduce the abbreviations ‘3!’ for the expression *exactly three* below. As we have observed before in discussing such quantifiers, the second alternative in (50b) is not entailed by the prejacent, but its negation is perfectly consistent with the prejacent. The meaning derived is (50c), which is true just in case Bill is a student that smokes.

(50)  
\*Exactly three students but Bill smoke  
\hspace{1em} a. \hspace{1em} EXH(A)(\lambda w.3!([\text{student}]^w – [\text{Bill}])([\text{smokes}]^w))  
\hspace{1em} c. \hspace{1em} ALT = \{ EXH(A)(\lambda w.3!([\text{student}]^w – [\text{Bill}])([\text{smokes}]^w)),  
\hspace{3em} EXH(A)(\lambda w.3!([\text{student}]^w – \emptyset)([\text{smokes}]^w)) \}  
\hspace{1em} b. \hspace{1em} Exactly three students who are not Bill smoke and it is not the case that exactly three students smoke.

The unacceptable case of non-monotone quantifiers cannot be ruled out by the condition that rules out *some*; it must be ruled out by uniqueness. The same applies to non-AA LDE quantificational determiners like *at most three* for the same reasons. Following von Fintel’s (1993) lead, we must make use of a further implication of uniqueness to rule out these cases. In the next section, we formalize uniqueness as another instance of exhaustification.

3.2.2. Unique minimality

The second layer of exhaustification indicates that the minimal exception set, denoted by the complement of *but*, is also unique. To implement this exhaustification, we need to consider another set of alternatives to the exception set. In this case, we must consider all alternatives to the exception set and not merely the subsets of the exception set. This is so, because we need the set denoted by the complement of *but* to be the absolutely unique minimal exception set. Furthermore, the alternative propositions considered by the second exhaustifier must already be exhaustified. That is, to say that we are comparing minimal exception sets and minimality is – as we have just shown in Section 3.2.1 – to be enforced by an instance of exhaustification. The result that we ultimately want for the truth conditions is of the shape in (52) for a structure like (51).

(51)  
\[
\begin{array}{c}
S \\
\text{DET} \\
\text{NP} \\
\text{but} \\
\text{C} \\
\end{array}
\]

(52)  
\[\text{EXH}(\text{ALT}-\text{C})(\lambda w.[\text{DET}][\text{NP}]^w – [\text{C}])[\text{VP}]^w) = 1\]
for all $S$, if $\text{exh}(\text{ALT}-S)(\lambda w. [[\text{Det}]](\![NP]\wedge S)(\![VP]\wedge)) = 1$, then

$$\text{exh}(\text{ALT}-C)(\lambda w. [[\text{Det}]](\![NP]\wedge [C])(\![VP]\wedge)) = \text{exh}(\text{ALT}-S)(\lambda w. [[\text{Det}]](\![NP]\wedge S)(\![VP]\wedge))$$

(53) Abbreviation for (52)

For any set of individuals $X$,

$$\text{ALT}-X = \{\lambda w. [[\text{Det}]](\![NP]\wedge Y)(\![VP]\wedge) : \text{but}(Y) \in \text{ALT}(\text{but}(X))\}$$

$$= \{\lambda w. [[\text{Det}]](\![NP]\wedge Y)(\![VP]\wedge) : Y \subseteq X \}$$

(cf. (46))

The prejacent, that is the basic unexhaustified statement $\text{Det} NP \text{ but } C VP$, makes reference to a specific set of exceptions $[C]$. The first instance of exhaustification, which enforced minimality, then makes reference to a set of alternative propositions that depend on the alternatives to $[C]$ triggered by the exceptive marker but. Recall that but invokes the set of subsets of $[C]$ as alternatives to it. The idea to be pursued here is that there is a second level of exhaustification, which enforces uniqueness, that makes reference to alternative propositions and to alternative alternative sets for the first minimality-inducing instance of exhaustification. That is, in (52), the set of alternatives against which the first layer of exhaustification happens must vary with the alternatives to the prejacent’s exception set that are being quantified over. How can this be accomplished technically? We need to consider two different sets of alternatives to the complement of but. I suggest that this can be accomplished if the exceptive marker lexically introduces second order alternatives. Below I will clarify what I mean by second order alternatives.

3.2.3. Fox (2007)

First, it will be worthwhile to discuss the inspiration for this account. Fox (2007) proposes that the free choice reading of disjunction derives from the application of recursive exhaustification. In an unembedded environment disjunction has its basic truth-functional meaning (54b) supplemented with a scalar implicature (54c) that not both of its disjuncts are true.

(54) a. Bill ate cake or ice cream.

b. Truth conditions: $C \lor I$  

c. Implicature: $\neg(C \land I)$  

(C: Bill ate cake, I: Bill ate ice cream)

Fox proposes that this implicature arises from the presence of an exhaustifier taking a set of scalar alternatives (e.g., (55)) as one of its arguments. Fox proposes, following Sauerland, that the alternatives to a disjunction include each of the disjuncts.

(55) $\text{ALT}(C \lor I) = \{C \lor I, C, I, C \land I\}$

In order to make consistent use of such an alternative set for disjunction, it is necessary to use a different version of the exhaustifier from what we have used so far. Fox introduces the notion of innocent exclusion in the semantics of exhaustification. An alternative to the prejacent is innocently excludable if and only if it is a part of every maximal set of alternatives that can be negated and remain consistent with the prejacent. Formally, it
looks like (56). ‘I-E(p,A)’ stands for the set of propositions innocently excludable in A with respect to p.

\[(56)\]
\begin{enumerate}
\item \(A^* = \{-p: p \in A\}\)
\item \(I-E(p,A) = \cap \{B: B\) is a maximal set in A s.t. \(B \cap \{p\}\}\)
\item \[\forall q \in I-E(p,A) \rightarrow q(w) = 0\]
\end{enumerate}

In the case of disjunction as in (54), the only innocently excludable proposition among the scalar alternatives (55) is the conjunctive alternative. Neither of the individual disjuncts is innocently excludable. While each can be negated consistently with the prejacent, they cannot both be denied consistently with the prejacent. Hence neither belongs to all the maximal sets of alternatives that can be negated consistently with the prejacent disjunction. In this way, this analysis derives the normal scalar implicature for unembedded disjunction.

The power of this analysis comes when disjunction is embedded under a modal. Consider for example (57). This sentence carries a free choice implication to the effect that Bill may have cake and Bill may have ice cream. This implication does not follow from the basic truth conditional meaning of this statement, (57b).

\[(57)\]
\begin{enumerate}
\item Bill may have cake or ice cream.
\item \(\Box(C \lor I)\)
\end{enumerate}

Applying one instance of exhaustification only yields a simple scalar implication of the usual kind, cf. (58).

\[(58)\]  
\[\text{EXH}_{\text{ALT}}(\Box(C \lor I)) \rightarrow (C \lor I) = \Box(C \lor I) \land \neg \Box(C \land I)\]

However, adding another layer of exhaustification yields the sought after free choice inference. The alternatives to the embedded exhaustified statement are as in (59b). I will leave it to the reader to determine that the second and third alternatives in the set (59b) have the meanings in (59c) and are both innocently excludable.

\[(59)\]
\begin{enumerate}
\item \(\text{EXH}_{\text{ALT}}(\text{EXH}_{\text{ALT}}(\Box(C \lor I)) \rightarrow (C \lor I)) \rightarrow (C \lor I)\)
\item \(\{ \text{EXH}_{\text{ALT}}(\Box(C \lor I)) \rightarrow (C \lor I), \text{EXH}_{\text{ALT}}(\Box(C \lor I)) \rightarrow C, \text{EXH}_{\text{ALT}}(\Box(C \lor I)) \rightarrow I, \text{EXH}_{\text{ALT}}(\Box(C \lor I)) \rightarrow (C \land I)\}\)
\item \(\text{EXH}_{\text{ALT}}(\Box(C \lor I)) \rightarrow C = \Box(C \neg \rightarrow I); \text{EXH}_{\text{ALT}}(\Box(C \lor I)) \rightarrow I = \Box(I \neg \rightarrow C)\)
\end{enumerate}

Both of these alternatives then will be negated by the second instance of exhaustification. From this derives the implication that Bill is allowed to have cake if and only if he is allowed to have ice cream, cf. (60). This implication together with the prejacent, which says that Bill is allowed to have ice cream or cake, entails that he is allowed to have ice cream and he is allowed to have cake. This is the free choice inference we sought to derive.

\[(60)\]
\[\neg(\Box C \land \rightarrow I) = \Box C \rightarrow I\]
b. \( \neg(\Box I \land \neg \Box C) = \Box I \rightarrow \Box C \)

c. Conjunction of (a) and (b): \( \Box C \leftrightarrow \Box I \)

This is an important result and establishes the usefulness of recursive exhaustification. I intend to use a similar kind of recursive exhaustification in the case of but-EPs. It is important, however, to point out some of the differences between Fox’s approach and the one I propose. First, I will not use an exhaustifier whose semantics is based on innocent exclusion. Innocent exclusion prevents an exhaustifier from deriving a meaning inconsistent with the prejacent. For an analysis based on von Fintel (1993) to work, exhaustification must sometimes lead to contradiction. Second, notice that under Fox’s account, when the second exhaustifier is introduced, alternatives to the alternative set fed to the first exhaustifier are not considered. Crucially, \( \Box C \) is assessed relative to the alternative set for \( \Box (C \lor I) \). This is necessary. Otherwise, \( \Box I \) would not be an alternative to \( \Box C \). As will become clear below, I must assume that the alternative set utilized by the first instance of exhaustification varies with the alternatives to the ‘focus,’ in this case the exception set. This will be made clearer in the next section.

3.2.4. Second-order alternatives

In this section I lay the groundwork for implementing the recursive exhaustification approach to but-EPs. Suppose that when an expression introduces alternatives, it creates an ordered pair of a plain meaning and a set of alternatives. On this view, an exhaustifier then takes such pairs of a proposition and set of propositions, (61), as input and returns a proposition.

(61) \(<p, \text{ALT}(p)>\)

This is partially a departure from the picture I have presented above where the connection between the exhaustifier and the set of alternatives associated with its argument is less direct. Before I suggested that the exceptive marker but conventionally introduces alternatives and the exhaustifier picks them up through anaphora – the exhaustifier has a contextually determined set of propositions as an argument. I am now suggesting a more direct connection between the alternative introducing expression and its exhaustifier. The presence of an alternative-trigger creates an ordered pair of meanings – a plain meaning and a set of alternatives; some operators like exhaustifiers operate on both coordinates of meaning. I am furthermore suggesting now that but introduces second-order alternatives. By this I mean that but lexically introduces alternatives to pairs of plain meanings and alternatives. In other words, a but-EP sentence denotes a pair of (i) a pair of a proposition and a set of propositions and (ii) a set of alternative pairs as illustrated in below in (62). For example, you compare Det NP but Bill VP, to Det N but X VP, where X is a subset of \{Bill\}, and then compare that pair to Det N but Joe, Al, Pete VP, which is in turn being compared to Det N but Y, where Y is a subset of \{Joe/Al/Pete\}, etc.

(62) \(<<p, \text{ALT}(p)>, \text{ALT}(p, \text{ALT}(p)>>\)
In particular, at the type $<<e,t>,<e,t>>$ level of the but-EP, the pair would look as below, where $C$ is the complement of but as before.

\[(63) \quad < \langle \text{but} \rangle([C]), \{ \langle \text{but} \rangle(S) : S \subseteq [C] \} >, \]
\[\{ \langle \text{but} \rangle(R), \{ \langle \text{but} \rangle(S) : S \subseteq R \} : R \in D_{<e,t>} \} > \]

So, but introduces first order alternatives to the plain meaning and also second order alternatives to the plain meaning and its alternatives. The idea then is that these alternatives must be exploited by exhaustifiers in order to ultimately arrive at a propositional meaning. Exhaustification is an operation that applies to a pair of a proposition and a set of alternative propositions. So, first an exhaustifier must apply to the first member of the pair (a pair itself) and pointwise to the pairs in the set that makes up the second member. Then the second exhaustifier applies to the resulting pair of a proposition and set of propositions.

\[(64) \quad <\text{p}_C, \text{ALT}(p_C)>, \{\text{p}_S, \text{ALT}(p_S) : S \in D_{<e,t>} \} > \]
\[<\text{EXH}(p_C, \text{ALT}(p_C)), \{ \text{EXH}(p_S, \text{ALT}(p_S)) : S \in D_{<e,t>} \} > \]
\[\text{EXH}(\text{EXH}(p_C, \text{ALT}(p_C)), \{ \text{EXH}(p_S, \text{ALT}(p_S)) : S \in D_{<e,t>} \} ) \]

3.3. Recursive exhaustifier: $\text{EXH}_R$

I formalize the process of recursive exhaustification in a single exhaustifier $\text{EXH}_R$ which performs the operations sketched in (64). Formally we may assume that but carries a feature that must be checked locally by $\text{EXH}_R$. We may define this new exhaustifier as in (65) below.

\[(65) \quad \text{For a proposition p, set of propositions } \Pi \text{ and a set of proposition-set of proposition pairs } \Sigma: \]
\[\text{EXH}_R(<p, \Pi>, \Sigma) = \text{EXH}<\text{EXH}(p, \Pi), \{ \text{EXH}(\sigma) : \sigma \in \Sigma \} > \]

In order to maintain the account of the incompatibility of but-EPs with associate quantifiers headed by LUE determiners, we need to impose a felicity condition on occurrences of the recursive exhaustifier. Recall that the suggested condition required that there be some alternative propositions that are eliminated by the exhaustification. I encode this condition now as below.

\[(66) \quad \text{Condition: In (65), } \Pi \text{ must include some propositions not entailed by p.} \]
With an explicit definition in place, let’s reconsider a simple example, (67a). This sentence has the structure in (67b), with the covert operator EXHR taking propositional scope. The presence of EXHR is required by the feature specification of the exceptive marker but.

(67)  a. No boy but Bill runs.

```
EXHR
  no
  runs
  boy but[EXH] Bill
```

The complement of the but-EP denotes a pair of (i) a pair of a proposition and a set of propositions and (ii) a set of pairs of a proposition and a set of propositions, as shown in (68).

(68)<< λw.[no]([[boy] w– {Bill}]([[runs] w)), {λw.[no]([[boy] w–S]([[runs] w]): S⊆{Bill}}>,
{<λw.[no]([[boy] w–R]([[runs] w)), {λw.[no]([[boy] w–S)([[runs] w]): S⊆R} >: R∈D<e,t>}>)

This pair serves as the argument of the recursive exhaustification operator. So, the first coordinate of (68) becomes the proposition that Bill is a minimal exception to the statement ‘no boy runs’. This means that Bill is a boy and does run. The second coordinate becomes a set of propositions each of which says that an alternative set is a minimal exception to the generalization, see (70).

(69)  EXHR (68)=
EXH<EXH(λw.[no]([[boy] w–{Bill}]([[runs] w)),
{λw.[no]([[boy] w–S)([[runs] w]): S⊆{Bill}}),
{EXH(λw.[no]([[boy] w–R]([[runs] w)), {λw.[no]([[boy] w–S)([[runs] w]): S⊆R} >: R∈D<e,t>}>)

(70)  EXH<λw.no boy in w but Bill runs in w,{λu.no boy in u but C runs in u:C≠{Bill}} >

Now, because no happens to be a determiner that guarantees unique minimal exceptions, the truth of the first coordinate in (70) actually entails the falsity of every alternative proposition that is a member of the second coordinate, except the proposition itself. So, all of the alternatives are excluded by the exhaustifier. But since their falsity was already entailed by the first coordinate, in this case, the second layer of exhaustification has no effect.

Now we must turn to one of the cases that motivated the inclusion of a second layer of exhaustification. Recall that, unlike no, not all determiners guarantee that a minimal exception is unique. Non-AA LDE quantifiers and non-monotone determiners allow non-unique minimal exceptions. Quantifiers headed by these determiners cannot felicitously host a but-EP. So, our goal is to rule out the combination through our new approach involving EXHR. Below I work through the case of a non-monotone determiner, exactly three. For convenience, I again abbreviate the expression exactly three as ‘3!’.
a. *Exactly three boys but Bill run.

b. EXHR

\[3!\] boys but Bill

Again, the denotation of the complement of EXHR can be found in (72) and the result of recursive exhaustification in (73). The exhaustified first coordinate is the proposition that Bill is a minimal exception to the statement that exactly three boys run. This implies that exactly four boys run and that Bill is one of them. The set of exhaustified alternatives contains propositions stating that other sets constitute minimal exceptions to the generalization that exactly three boys run. None of these alternatives is entailed by the first coordinate – again, except for the proposition itself. But not all of them are entailed to be false by it either, unlike the case in (69). So, the exclusion of these alternatives is contentful.

\[(72)\] \(<< \lambda w. [3!][[\text{boy}][w] \sim \{\text{Bill}\}](\text{runs}[w]), \{\lambda w. [3!][[\text{boy}][w] \sim S](\text{runs}[w]): S \subseteq \{\text{Bill}\}\}, \{\lambda w. [3!][[\text{boy}][w] \sim R](\text{runs}[w]), \{\lambda w. [3!][[\text{boy}][w] \sim S](\text{runs}[w]): S \subseteq R\}>, R \in D_{<e,t}>\>

\[(73)\] \(\text{EXHR}(71)=\)

\(\text{EXH}<\text{EXH}(\lambda w. [3!][[\text{boy}][w] \sim \{\text{Bill}\}](\text{runs}[w]), \{\lambda w. [3!][[\text{boy}][w] \sim S](\text{runs}[w]): S \subseteq \{\text{Bill}\}\}, \{\lambda w. [3!][[\text{boy}][w] \sim R](\text{runs}[w]), \{\lambda w. [3!][[\text{boy}][w] \sim S](\text{runs}[w]): S \subseteq R\}>, R \in D_{<e,t}>\>

\[(74)\] \(\text{EXH} (\lambda w. [3!][[\text{boy}][w] \sim \{\text{Bill}\}](\text{runs}[w]), \{\lambda w. [3!][[\text{boy}][w] \sim S](\text{runs}[w]): S \subseteq \{\text{Bill}\}\}) = \text{Bill is a boy and Bill and exactly three other boys run (Abbreviated: ‘3+Bill boys run’)}\)

\[(75)\] \(\text{EXH}<3+\text{Bill boys run}, \{3+C \text{ boys run: } C \neq \{\text{Bill}\}\}>\)

This additional content, however, leads to a contradiction. While no particular alternative set is entailed to be a minimal exception, it is entailed that there must exist three other minimal exception sets. So, if exhaustification requires that all alternative propositions are false, since they are not entailed, this contradicts the exhaustified first coordinate of (75).

Thus, we are able to rule out the co-occurrence of but-EPs with quantifiers headed by determiners that allow multiple minimal exceptions to a generalization. The second layer of exhaustification, which enforces the uniqueness of minimal exceptions, leads to contradictions in these cases. In the other cases, the second layer of exhaustification is harmless. The combination of LUE determiners and but-EPs is ruled out by a presupposition at the first layer, since no alternative propositions are eliminated. In the case of AA, LDE determiners every and no, the correct semantics is derived at the first level and the second level’s contribution is redundant.
4. The scope of $\text{EXH}_R$

A question arises as to what are the possible scopes for the exhaustifier. The answer affects the possible interpretations of $\text{but}$-EPs and, hence, its distribution. Recall that in order to account for uses of $\text{but}$-EPs with $\text{any}$, it must be possible for $\text{EXH}_R$ to take scope at a position that is not local to the EP associate. In other words, $\text{EXH}_R$ need not occur at the first scope site c-commanding the $\text{but}$-EP’s associate quantifier. For example, in principle, $\text{EXH}_R$ could take scope below $\text{no professor}$, but must be allowed to scope above it for the story in section 2.2 to work.

(76)  $\text{EXH}_R$ No professor saw any student but$_{[+\text{EXH}]}$ Bill

4.1. Constraints on scope for $\text{EXH}_R$

We want to know now whether there are any other cases in which non-local scope is allowed. In (77), there are two possible scopes for $\text{EXH}_R$, (77a) and (77b).

(77)  Every professor saw no student but Bill  
  a. Every professor$_x$ [ $\text{EXH}_R$ no student but Bill$_y$ [ $x$ saw $y$ ] ]  
     ‘Bill is a student, every prof saw Bill, no other student was seen by any prof.’  
  b. $\text{EXH}_R$ Every professor$_x$ [ no student but Bill$_y$ [ $x$ saw $y$ ] ]  
     ‘Bill is a student, some prof saw Bill, no other student was seen by any prof.’

Reading (77a) is available. The weaker reading (77b), however, which does not entail that every professor saw Bill, is not available. We should also keep in mind other possible relative scope of the quantifiers – even though it is often difficult for negative quantifiers like $\text{no student}$ to scope over universals.

(78)  $\text{EXH}_R$ [ no student but Bill$_y$ [ every professor$_x$ [ $x$ saw $y$ ] ] ]  
     ‘Bill is a student, every prof saw Bill, no other student was seen by every prof.’

(77a) asymmetrically entails both (77b) and (78) Note also that (77b) and (78) are independent. So, the judgment is solid that (77b) is unavailable. The tentative conclusion, then, seems to be the following.

(79)  $\text{but}$-EPs require their exhaustifier ($\text{EXH}_R$) to be as local as possible.

A possible explanation for the perceived markedness of (77b) might be a preference for placements of the exhaustifier that yield stronger readings. Several researchers have recently proposed such a constraint on the placement of exhaustifiers. I include one below.

(80)  Let $\phi$ be a logical form. Let $\phi$’s competitors be all the LFs that differ from $\phi$ only with respect to where exhaustivity operations occur. Then everything else being
equal, \( \phi \) is dispreferred if one of its competitors is stronger. 

(Chierchia, Fox, Spector 2008)

Assuming that \( \text{EXH}_R \) must take scope very locally may help to explain some aspects of but-EPs distribution. For example, even though a consistent meaning could be derived for (81a), if \( \text{EXH}_R \) took scope over \textit{no professor}, such a meaning is not available. This could be accounted for if \( \text{EXH}_R \) was forced to scope below \textit{no professor}, as in (81b).

\[(81) \quad \begin{align*}
\text{a. } & \text{No professor saw a student but Bill.} \\
\text{b. } & \text{No professor}_x \ [\text{EXH}_R \text{ a student but Bill}_y [x \text{ saw y}]]
\end{align*}\]

But, of course, this leaves us with a question. Why is the exhaustifier \( \text{EXH}_R \) allowed to take wider scope when the EP is attached to a DP headed by \textit{any}?

4.2. Relativized minimality

Following Chiechia’s (2006, 2010) approach to intervention, I assume that the exhaustifier triggered by \textit{but} belongs to the same system as the exhaustifier that Chierchia (2004) uses to account for the distribution of \textit{any}. Because they belong to the same class of operators, crossing dependencies are not allowed. An exhaustifier is not allowed to skip an (unchecked) \( \text{EXH} \)-feature. Nesting dependencies are allowed.

\[(82) \quad \begin{array}{c}
\text{EXH}_R \quad \text{EXH} \\
\text{No professor saw any}_{+\text{EXH}} \quad \text{student but}_{+\text{EXH}} \quad \text{Bill}
\end{array}\]

\[(83) \quad \begin{array}{c}
\text{EXH} \\
\text{[ No professor}_x \quad \text{EXH}_R \quad [\text{any}_{+\text{EXH}} \quad \text{student but}_{+\text{EXH}} \quad \text{Bill}_y [x \text{ saw y}]]]
\end{array}\]

In this way we can reduce the problem of the scope of \textit{but}’s exhaustifier to the problem of \textit{any}’. Licensing of \textit{any} requires its exhaustifier to scope above its negative licenser. Relativized minimality forces \textit{but}’s exahustifier to scope above \textit{any}’s. Obviously, for this to work the locality conditions on \textit{but}’s exhaustifier must be violable and ranked below the conditions on \textit{any}’s.

An additional prediction of this story is that an unlicensed \textit{any} should improve a \textit{but}-EP’s co-occurrence with a c-commanding non-NPI existential associate. In some cases this is a good prediction, in others it is less clear, cf. (84) and (85).

\[(84) \quad \text{Mary doesn’t like a relative of any politician but Billy Carter.}\]

\[(85) \quad \text{Mary doesn’t know a person who owns any ferrets but Fred.}\]

At this point, this account remains a sketch of an approach to how to limit \textit{but}-EPs to associating with NPIs in the scope of DE operators.

5. Conclusion
In this paper I have tried to argue for a parallel between the semantics/pragmatics of the exceptive marker *but* and that of polarity/free choice items. The idea is simple. The basic semantics of EM *but* is that of set subtraction; it removes the individuals denoted by its complement from the restrictor of the quantifier it is attached to. In addition, *but* is an alternative-introducing expression. In fact, I argue that a *but*-EP introduces second order alternatives, alternatives to a proposition p and to p’s set of alternatives. These second-order alternatives then are predictably exploited by a particular exhaustifier, which I labeled $EXH_R$. This exhaustifier applies recursively, imposing minimality on the semantics of *but*-EPs at one level and then imposing uniqueness at the second level. The analysis finds antecedents in the work of Fox (2007) and Chierchia (2006, 2010).

There are still areas of the account that need work. In Section 4, we looked in a preliminary way at possible scopes that $EXH_R$ could take with respect to the surface position of *but*-EPs. A fuller understanding of the constraints on scope awaits further research. So does our precise understanding of why *but*-EPs can associate with any in the scope of a DE operator, but not with non-NPI indefinites like a.

References

Gajewski, Jon. 2002. L-analyticity and natural language. Ms. MIT.
1 The complement of a but-EP need not be referential; it could denote a generalized quantifier. I will not address this complex issue here, but see Moltmann (1995) and von Fintel (2000) for discussion.
2 In particular, making nobody but Al Strawson-DE will not help. This would predict that only Al and nobody but Al should have the same licensing properties, but they do not.
3 Of course, an alternative is the view of Dayal (1998, 2004), who argues that free choice any is in fact a universal quantifier. If this is true, it is compatible with von Fintel’s account.
4 Note that I have added a presupposition in (37) to the effect that every member of the exception set belongs to the restrictor. This is justified by projection facts.
5 Alternatively, we might make the analogy between but and any and say that but-EPs carry an implicit restriction. We could then consider alternatives to this implicit restriction which would then be intersected with the denotation of the complement of but. This would have the same effect as restricting the set of alternatives to the exception set to subsets. At this time, however, I can see no reason to assign the but-EP its own implicit restrictor.
6 Chierchia’s condition is a bit different, forbidding non-entailed alternatives that are not eliminated – a possibility in his system.
7 Here I follow the lead of Chierchia (2010), who formalizes recursive exhaustification with an operator OR.