More on quantifiers in comparative clauses

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1. Introduction

Focus: interpretation of DP-quantifiers in clausal comparatives.

(1) Mary is taller **than every boy is**

Goal: endorse the semantics for clausal comparatives given below:

(2) E(xistential)-Theory
For A an individual, P a scalar predicate and Q a DP quantifier
A **is** P-**er than** Q **is** is True iff
$\underbrace{\{d: \text{A is d-P}\}}_{\text{MC}} \cap \underbrace{\{d: \text{Q is not d-P}\}}_{\text{CC}} \neq \emptyset$ (cp. Seuren 73)

I assume throughout that gradable adjectives are monotone.

(3) a. a function of type <d,<e,t>> is monotone iff
$\forall x \forall d \forall d'[f(d)(x)=1 \ \& \ d'<d \rightarrow f(d')(x)=1]$
b. [[**tall**]] = $\lambda d.\lambda x.$HEIGHT$(x) \geq d$ (cf. Heim 00)

We will compare the E-theory with a recent successful alternative:

(4) M(aximality)-Theory
For A an individual, P a scalar predicate and Q a DP quantifier
A **is** P-**er than** Q **is** is True iff
max({d:A is d-P})∈ {d: Q is not d-P} (cp. Heim 06[1], Schwarzschild 04,08)

(5) $\max(D) := \iota d[d \in D \ \& \ \forall d' \in D \ d' \leq d]$

(6) **Mary is taller than Bill is** is True iff
a. E-theory: {d: Mary is d-tall}∩{d: Bill is not d-tall}≠∅
b. M-theory: Mary's height ∈ {d: Bill is not d-tall}

1.1 Basic Facts about Qs in CC

(7) a. Q is upward entailing (UE) iff for all sets A, B s.t. A ⊆ B, Q(A) ⇒ Q(B)
b. **Mary is taller than some boy is.**
E-theory: {Mary is d-tall}∩{d:some boy is not d-tall}≠∅ ✓
M-theory: Mary's height ∈ {d: some boy is not d-tall} ✓

[1] Equivalence with CC in Heim 06: when F is monotone and has a maximum, {d: not F(d)} = {d: d>max(F)}

(8) a. Q is downward entailing (DE) iff for all sets A, B s.t. A ⊆ B, Q(B)⇒Q(A)
b. ***Mary is taller than no boy is.**
E-theory: {Mary is d-tall}∩{d:no boy is not d-tall}≠∅ ✓**[tautology]**
M-theory: Mary's height ∈ {d: no boy is not d-tall} **[wrong]**

(9) a. Q is non-monotonic (NM) iff Q is neither UE nor DE
b. Mary is taller than exactly 2 boys are.
E-theory: {Mary is d-tall}∩{d:exactly 2 boys are not d-tall}≠∅ **[too weak]**
M-theory: Mary's height ∈ {d: exactly 2 is not d-tall} ✓

1.2 Proposal
E-Theory is superior to M-Theory:
a. The two theories are equivalent when Q is upward entailing.
b. M-Theory fails with downward entailing Qs; E-Theory rules them out as tautologies.
c. M-theory succeeds with non-monotonic quantifiers like **exactly 2**; E-Theory appears to fail, yielding weak truth-conditions. Independently motivated approach to **exactly** (Landman 98, Krifka 99) saves E-Theory.
d. Other cases of non-monotonic Qs favor E-Theory. Some remaining problems for the E-theory are discussed.

2. Downward Entailing Quantifiers (DE-Qs)

DE-quantifiers are (in general) unacceptable in comparative clauses.

(10) a. *Mary is taller than no boys are.
b. *Mary is taller than few boys are.
c. *Mary is taller than fewer than eight boys are.
d. *Mary is taller than not every boy is.
e. ?Mary is taller than at most three boys are.

2.1 Standard approach: von Stechow 84, Rullmann 95

Von Stechow 84/Rullmann 95 propose that the *than*-clause denotes the maximum of a set of degrees. I follow Heim's 00 implementation of this idea, in assuming that degree predicates are monotone functions of type <d,<e,t>>.[2]

(11) Bill is taller than Fred is
LF: [-er [wh$_1$ Fred is t$_{1,d}$ tall]]$_{2,d}$ Bill is t$_{2,d}$ tall
TC: max(λd.Bill is d-tall) > max(λd.Fred is d-tall)

[2] von Stechow 84 and Rullmann 95 use an 'exactly' interpretation of degree predicates, but the need to scope out *every*, noted below, is the same.

This theory provides an immediate explanation of the unacceptability of DE-Qs in comparative clauses.

(12) *Bill is taller than no girl is
LF: [-er [wh$_1$ no girl is $t_{1,d}$ tall]]$_{2,d}$ Bill is $t_{2,d}$ tall
TC: max(λd.Bill is d-tall) > max(λd.no girl is d-tall)

(13) {d: no girl is d-tall} has no maximum; (12) is thus undefined.

This approach fails for many other quantifiers. To get the correct truth conditions in these other cases, the quantifiers must scope out.

(14) Bill is taller than every girl is.
max(λd.Bill is d-tall) > max(λd.every girl is d-tall)
the height of the shortest girl!

(15) every girl$_x$ Bill is taller than x is.

The same point applies to **most boys**, **Bill and Fred**, and **exactly 2 girls**. If these quantifiers are allowed to scope out, one needs a principled reason to block **no girl** from scoping out giving the coherent (16) as LF for (12):

(16) no girl$_x$ Bill is taller than x is.

<u>Note:</u> von Stechow 84 and Rullmann 95 argue that DE-Qs are not the only Qs that scope <u>under</u> *max*/negation.

(17) Mary is taller than <u>any</u> boy is
a. Mary's height $\in$ {d: <u>not</u> [a boy is d-tall]}
b. #Mary's height $\in$ {d: a boy is <u>not</u> d-tall]} (wrong meaning)

(18) Mary is taller than Bill or Fred is.
Mary's height $\in$ {d: <u>not</u> [Bill or Fred is d-tall]}

There is some reason to think these are instances of <u>free choice</u>. Plausible generalization:

(19) DP quantifiers scope over max/negation in the *than*-clause.
(related to Kennedy 1999/Heim 2000)

2.2 M-Theory on DE-Qs in CC

The M-Theory is designed to give a better account of Qs in CC. Under M-Theory there is no need to scope **every boy**, **most boys**, **Bill and Fred**, and **exactly 2 girls** out of the *than*-clause (cf. Schwarzschild & Wilkinson 02, Heim 06).

(20) Mary is taller than every boy is.
M-theory: Mary's height $\in$ {d: every boy is not d-tall}

Despite this success with previously problematic cases of UE quantifiers, M-theory fails with DE-Qs.[3]

(21) a. *Bill is taller than no girls are.
b. M-theory: Bill's height $\in$ {d: no girls are not d-tall}
"Bill is at most as tall as the shortest girl"

This is not a particular failure with **no girl** but extends to all DE-Qs. When Q is DE, CC is downward closed.

(22) If Q is DE, CC is DC.
(23) A set of degrees D is downward closed (DC) iff
$\forall$d,d'[d $\in$ D and d'<d $\rightarrow$ d' $\in$ D]

Thus, when CC contains a DE-Q it imposes a maximum on the height of the subject. Another example:

(24) *Bill is taller than not every girl is.
M-Theory: Bill's height $\in$ {d: not every girl is not d-tall}
"Bill is at most as tall as the tallest girl"

2.3 E-theory on DE-Qs in CC

E-theory inherits M-theory's success with quantifiers but also offers an account of DE-Qs. E-theory predicts that a CC containing a DE-Q yields trivial truth conditions.

Point one: E-Theory inherits M-theory's advantages with UE-Qs. In fact, E-theory and M-theory are equivalent when Q in CC is UE.

(25) MC is DC and MC has a maximum.
(26) If Q is UE, CC is UC.

[3] Schwarzschild 04 & Heim 06 use a flexible scope *max* in CC. If that *max* scoped over a DE-Q in CC, there would be a presupposition failure. The question of where *max* scopes is the same as the scope issue for von Stechow 84.

(27) A set of degrees D is upward closed (UC) iff
$\forall d,d'[\, d \in D \text{ and } d'<d \rightarrow d' \in D]$

(28) For sets of degrees D,D' where D is DC and has a maximum and D' is UC, $\max(D)\in D'$ iff $D\cap D'\neq\varnothing$

Hence, **every boy**, **most boys** and **Fred and Bill** are handled without scoping these Qs out.

Point two: E-theory predicts that when Q is DE the comparative statement is trivially true.

This is first suggested in von Stechow's 84 (p.34) discussion of how Seuren 73 might handle (29):[4]

(29) a. *Sue is smarter than neither Bill nor Mary is
b. (∃d)[Sue is d-smart & ~~(Bill or Mary is d-smart)] (von Stechow 84 (101))

[Note Seuren 73 differs from E-theory in scope of negation; but this is irrelevant to the triviality account.]

As observed above, MC is always DC and when CC contains a DE-Q it is DC as well. These two observations do not quite guarantee that when Q is DE a comparative is always true, under E-Theory. The E-theory requires that the intersection of MC and CC is not empty.

It is plausible to assume that MC is always non-empty:

(30) [[**tall**]] = λd.λx: <u>x ∈ domain(HEIGHT)</u>.HEIGHT(x) ≥ d

Note: on some scales the measure assigned to an individual might be **0**.

I will stipulate that CC is non-empty, as well.[5] This is necessary, though in most cases difficult to distinguish from existence presuppositions of Q.

(31) {d: not every student is not d-tall} (= {d: some student is d-tall})

(32) | Assumption: CC is presupposed to be non-empty. |

Given (22), (30), and (32), it follows that when Q is DE and the comparative sentence is defined, it will always be true.

(33) The intersection of any two non-empty DC sets of degrees is non-empty. ○

[4] See also Kennedy 99 pp. 60-1 contra tautology analysis of ban on scope over DE-Q in main clause.
[5] This issue doesn't arise in von Stechow's example (29) because CC contains referring expressions.

(34) *Fred is taller than no student is
E-theory: {d: Fred is d-tall}∩{d: no student is not d-tall}≠∅
({d: every student is d-tall})

<u>Summarizing:</u>
+When Q is DE, M-Theory incorrectly predicts a maximum for the height of the subject.
+Under the assumption that CC is non-empty, E-Theory predicts that when Q is DE a comparative sentence has trivial truth conditions.
+I propose triviality can explain unacceptability in some cases.

Appendix to Section 2

A2.1 Are trivial sentences unacceptable?

Precedent

Model analysis: von Fintel 93 on exceptives (see also Barwise & Cooper 81).

(35) a. all/no/*some/*many student <u>but Bill</u> failed the exam

(36) [Det NP [[**but**]] S] VP =True iff
$VP\in Det(NP\text{-}S)\ \&\ \forall X(VP\in Det(NP\text{-}X) \rightarrow S \subseteq X)$

Under von Fintel's account, the ungrammatical cases come out as trivial. For example, it is possible to deduce from von Fintel's semantics that substituting any left upward monotone Q in (36) yields a contradiction.

Principle

Clearly, not all trivial sentences are unacceptable. So, we need criteria for distinguishing the good from the bad. For attempts along this line see Chierchia 1984, Fox and Hackl 2006. The present analysis would fit well with the latter's ideas. (For related discussion see Ladusaw 86.)

A2.2 Possible argument against using *max* to rule out DE Qs in CC

Rullmann 95 extends this maximality account to 'negative island' effects in degree questions.

(37) a. How hard did Bill work?
b. *How hard did Bill not work?

Fox & Hackl 06 observe that the effect disappears if the right modal is inserted.

(38) a. How hard is Bill not allowed to work?

b. How hard is Bill certain not to work?

(39) a. *Fred worked harder than Bill is not allowed to.
b. *Fred worked harder than Bill is certain not to.

3. Non-monotonic Quantifiers (NM-Qs)

The case of NM-Qs provides crucial motivation for M-theory (cf. Heim 06, Schwarzschild 08). M-theory gets **exactly two** just right; E-theory yields truth conditions that are too weak.

(40) Mary is taller than exactly two boys are.
#E-theory: {Mary is d-tall}$\cap${d:exactly 2 boys are not d-tall}$\neq\varnothing$ **[too weak]**
"Mary is taller than at least two boys are"
M-theory: Mary's height $\in$ {d: exactly two boys are not d-tall}

(41) Mary is taller than 10 to15 boys are.
#E-theory: {Mary is d-tall}$\cap${d:10 to 15 boys are not d-tall}$\neq\varnothing$ **[too weak]**
"Mary is taller than at least ten boys are"
M-theory: Mary's height $\in$ {d: 10 to 15 are not d-tall}

In these cases, we see the comparative imposing a maximum on the height of the student. E-theory is not capable of imposing a maximum.

Exactly two boys can be viewed as the coordination of a UE-Q and a DE-Q

(42) exactly two boys $\approx$ two boys and not more than two boys

Note: M-theory's success with **exactly 2** is tied directly to its failure with DE-Qs.

E-theory on NM quantifiers

3.1 *Exactly*

We've seen E-theory can capture unacceptability of DE-Qs. In this section, I suggest its weakness with NM-Qs is only apparent.

(43) Mary is taller than exactly two boys are.
#E-theory: {Mary is d-tall}$\cap${d:exactly 2 boys are not d-tall}$\neq\varnothing$ **[too weak]**

Goals: 1) rule out this too weak reading. 2) derive the correct reading

Notice that, in context, UE-Qs can impose maximums by scalar implicature.

(44) Mary is taller than some of the boys are.
Implicature: Mary is not taller than all the boys are.

(45) Mary is taller than two boys are.
Implicature: Mary is not taller than three boys are.

I suggest, following Landman 98, that **exactly** triggers the obligatory application of implicature-generating mechanisms.

(46) Exactly two students danced with exactly two professors.

(47) Two students danced with two professors.

The mechanism I assume is Fox's 06 alternative-sensitive EXH. I mark the 'focus' of EXH with **bold**.

(48) exactly 2 students smoke
logical form: EXH[**2** students smoke]

(49) $\text{EXH}(p_{st})(A_{<st,t>})(w)$ iff $p(w)=1$ & $\forall q\in A[\, q(w)=1 \rightarrow p \Rightarrow q\,]$

Generally, the EXH triggered by **exactly** takes local scope:

(50) Every boy read exactly two books
a. Every boy$_x$ EXH[x read **2** books]
b. #EXH[every boy read **2** books] (unavailable meaning)
"Every boy read two books and not every boy read three books"

(51) Local Scope EXH
Mary is taller than exactly two boys are.
#{Mary is d-tall}$\cap${d: EXH **2** boys are not d-tall}$\neq\varnothing$ (wrong meaning)
"Mary is taller than (at least) two boys are."

Proposal: when EXH is triggered it must have an effect on truth conditions. In (51), it has no effect: (52)a and b are equivalent. This, I claim, rule out local scope, as in (51), and licenses wide scope for EXH, as in (53).

(52) a. {Mary is d-tall}$\cap${d: EXH **2** boys are not d-tall}$\neq\varnothing$
b. {Mary is d-tall}$\cap${d: 2 boys are not d-tall}$\neq\varnothing$

(53) Wide Scope EXH
Mary is taller than exactly two boys are.
EXH[{Mary is d-tall}$\cap${d: **2** boys are not d-tall}$\neq\varnothing$]

Note: the equivalence between (52)a and b holds only under the assumption that the CC {d: EXH[**2** boys are not d-tall]} is non-empty, which I have already suggested is presupposed.

Beck 08/ms. arrives at a similar conclusion about **exactly** for related but different reasons in an interval-based semantics for comparatives. (For related work on modals see Krasikova 2007.)

> **Summarizing:**
> +When a comparative entails a maximum for the subject, the maximum derives from the lexically triggered application of implicature-generating mechanism EXH (Landman 98, Fox 06).
> +The EXH is allowed to outscope the comparative when taking narrow scope yields no effect on truth conditions.

3.2 Other continuous NM-Qs.
This analysis can be extended to other quantifiers that can be analyzed as the conjunction of a UE-Q and a DE-Q (a continuous quantifier, cf. Keenan 96), where the upper bound is imposed by EXH.

(54) Q is continuous iff for all X,Y,Z if $X \in Q$ & $Z \in Q$ & $X \subseteq Y \subseteq Z$, then $Y \in Q$

An example is **10 to 15 boys** (**between 10 and 15 boys**).

(55) Mary is taller than 10 to 15 boys are.

(56) n **to** m NPs VP
logical form: ∃d∈[n,m] EXH [**d**-many NPs VP]
([n,m] is the closed interval from n to m)

The set of degrees (57)a is always an initial segment of (57)b.

(57) a. {d: ∃n∈[10,15]EXH[**n**-many boys are not d-tall]}
b. {d: ∃n∈[10,15][n-many boys are not d-tall]}

This licenses wide scope for EXH. The indefinite quantifier over numbers must then also scope out, perhaps by choice function.[6]

(58) ∃n∈[10,15][EXH[∃d[Bill is d-tall and **n**-many students are not d-tall]]]

3.3 Explicit conjunctions of UE and DE quantifiers
Notice that this solution does not allow just any NM-Q to impose a maximum in a comparative, only those whose upper bounds come from EXH.

[6] I assume that EXH cannot take an operator's O trace as focus when O is within EXH's scope, as this leads to vacuous quantification in the focus alternatives.

The explicit coordination of a UE and a DE quantifier does not get its upper bound from EXH.

(59) a. *Bill is taller than some girls but no boys are.
b.*Bill is taller than every boy but not every girl is.

(60) $\lambda P_{<e,t>}$.[[**some girls**]](P) = 1 and [[**no boys**]](P)=1

These sentences not only lack maximums, they are unacceptable.

(61) *Bill is taller than some girls but no boys are.
E-theory: ∃d[Bill is d-tall & some girls but no boys are not d-tall]

The E-theory's predicted meaning is equivalent to the sentence without the DE conjunct.[7] I suggest this trivial of contribution as the source of unacceptablity.

Conjunctions of determiners are predicted to have the same status.

(62) ??Bill is taller than some but not all the students are.
??Bill is taller than more than 2 but fewer than 7 students are.

Judgments on these vary; but for nearly everyone they are still degraded but an improvement over (59). I do not know why this is so.

M-theory cannot distinguish between kinds of NM-Qs.

Appendix to section 3:Other cases of EXH taking wide scope

A3.1 Exceptives
Gajewski 08 argues that connected exceptive phrases need to be analyzed in terms of a strengthening operator that takes clausal scope.

(63) Every student but Bill failed.
EXH[every student–**{Bill}** failed]

(64) Mary is taller than every student but Bill is.

When non-empty, (65)a is an initial segment of (65)b. So, wide scope is licensed for EXH, yielding (66).

(65) a. {d: EXH[every student–**{Bill}** is not d-tall]}
b. {d: every student–**{Bill}** is not d-tall}

[7] Again, under the presupposition that CC is non-empty.

(66) EXH[∃d[Mary is d-tall & every student–**<u>{Bill}</u>** is not d-tall]]

A3.2 *at most*

The DE expression **at most 3 boys** is generally judged to be better in CC than other DE quantifiers. Landman 98 and Krifka 99 both treat **at most** along the same lines as **exactly**. We might attempt a similar analysis:

(67) ?Bill is taller than at most three students are.

(68) **at most** n **boys** ≈ **0 to** n **boys**

(69) {d: ∃n∈[0,3][EXH[**<u>n</u>** students are not d-tall]]}

(70) ∃n∈[0,3][EXH[∃d[Bill is d-tall & **<u>n</u>** students are not d-tall]]]

4. Problematic Cases

Non-continuous NM Qs like **an even number of** NPs yield CCs with gaps.

(71) Bill is taller than an even number of students are.

(72) **an even number of** NPs VP
logical form: ∃n∈{m: m is even}[EXH[**<u>n</u>**-many NPs VP]]

(73) a. {d: ∃n∈{m: m is even}[EXH[**<u>n</u>**-many students are not d-tall]]}
b. {d: ∃n∈{m: m is even}[n-many students are not d-tall]}

The interval (73)a is the union of CCs for **exactly n students** where n is even. The M-theory does fine here; the E-theory faces a problem. It is not guaranteed that (73)a spans an initial segment of (73)b.

Hence comparatives containing (73)a and b are not equivalent. So, there is no reason to rule out narrow scope for EXH.

The equivalence does hold under the assumption that the measure function - in this case *height* - is injective. That is, all members of the domain are distinguished from each other in measure.

(74) ∃n∈{m: m is even}EXH[∃d[Bill is d-tall & **<u>n</u>**-many is not d-tall]]

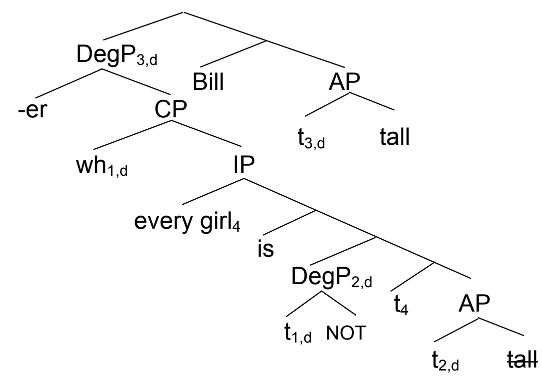
(75) Other non-continuous NM-Qs: exactly 3 or exactly 7 doctors, between 3 and 8 boys or exactly six girls, etc.

Conclusion
The weaker E-Theory is preferable to M-Theory.
- E-Theory and M-Theory equivalent for UE-Qs.
- DE-Qs yield trivial truth conditions under E-Theory. M-Theory predicts maximums.
- Some but not all NM-Qs impose maximums with wide scope EXH. E-Theory forces wide scope for EXH.
- Non-continuous NM-Qs handled by M-Theory & possibly E-Theory.

Appendix: LF syntax

I assume a Bresnan 73-style syntax:

Bill is taller than every girl is



Degree negation: [[NOT]] = $\lambda d.\lambda f_{<d,t>}.f(d)=0$ see Heim's 06 *little*
M-theory: [[**-er**]] = $\lambda P_{<d,t>}.\lambda Q_{<d,t>}.\max(Q)\in P$
E-theory: [[**-er**]] = $\lambda P_{<d,t>}:P \neq \emptyset.\lambda Q_{<d,t>}.P\cap Q\neq\emptyset$

The DegP embedded in the *than*-clause is capable of taking higher scope. This is apparently necessary for certain modals: *allow*, *have to*, *require* etc. For DPs it is not possible:

(76) <u>Heim/Kennedy Generalization</u>
If the scope of a quantificational DP contains the trace of a DegP, it also contains that DegP itself. (Heim 00)

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