Neg-Raising and Polarity 7/9/07

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1. Introduction

There is a class of sentence-embedding predicates in English and many other languages, called Neg-Raising predicates, with a peculiar property. When negated, these predicates imply a corresponding sentence in which the negation takes scope in the embedded clause. For example, intuitively, (1)a implies (1)b.

(1) a. Bill doesn't think that Mary is here.
b. Bill thinks that Mary is not here.

This property is striking since, given our best guess at the semantics of *think* outside of negative environments, its external negation should not entail its internal negation. And yet, native speakers clearly feel that (1)a implies (1)b. For sake of comparison, consider the non-NR predicate *say*. In this case, (2)b clearly does not follow from (2)a.

(2) a. Bill didn't say that Mary is here.
b. Bill said that Mary isn't here.

1.1 Approaches to Neg-Raising

The grammatical status of Neg-Raising (henceforth, NR) is controversial. The oldest idea about NR is that it is a syntactic operation (hence the name), cf. Fillmore (1963), Ross (1973), Prince (1976) a.o. Under this hypothesis, what distinguishes NR predicates (henceforth NRPs) is that they allow negation to be raised across them.

(3) a. Bill PRES __ think Mary is not here (NR)
b. Mary PAST __ say that Bill is not here (non-NR)

Doubt has been cast on the syntactic approach, most prominently by Horn (1978). One of the primary goals of this paper is to further develop an argument of Horn's against the syntactic approach.

Alternatively, NR has been conceived of as a semantic/pragmatic matter. From this perspective, one asks what is the status of the inference in (1). Is it grammatical, deriving from the lexical semantics of the predicate itself? Or is it extragrammatical, a Gricean implicature, perhaps? The evidence is mixed. In favor of the extragrammatical approach is the apparent defeasibility of the implication from (1)a to (1)b. Given the appropriate context, (1)b need not follow from (1)a. For example, given the context in (4)a, (5) does not intuitively follow from (4)b.

(4) a. Bill doesn't know who killed Caesar. Furthermore, Bill isn't sure whether or not Brutus and Caesar lived at the same time. So, naturally,

b. Bill doesn't think Brutus killed Caesar.[1]

(cp. Bartsch 1973)

(5) Bill thinks Brutus didn't kill Caesar.

Note that under the syntactic approach, this "defeasibility" is explained easily as a structural ambiguity.

Bartsch (1973) presents a simple and seductive approach to NR in the semantics/pragmatic vein. Arguing that there is no need for a syntactic operation of NR, Bartsch shows how the basic intuition about NR (the inference from (1)a to (1)b) can be derived from a pragmatic presupposition. Specifically, Bartsch proposes that NR predicates have whatever semantics we want to assign them, but in addition invoke an excluded middle presupposition. For example, Bartsch assigns the sentence (6) the truth conditions in (6)a and proposes that it invokes the excluded middle presupposition (6)b. ('B_a' stands for the set of worlds compatible with a's beliefs.)

(6) *a believes that p*
a. Truth conditions: $\forall w(w \in B_a \rightarrow w \in p)$
b. Presupposition: $\forall w(w \in B_a \rightarrow w \in p) \vee \forall w(w \in B_a \rightarrow w \notin p)$

The excluded middle presupposition says that either a believes p or a believes not-p. Now, given the standard assumption that presuppositions survive negation, we obtain the following results for the negation of (6).

(7) *a doesn't believe that p*
a. Truth conditions: $\neg \forall w(w \in B_a \rightarrow w \in p)$
b. Presupposition: $\forall w(w \in B_a \rightarrow w \in p) \vee \forall w(w \in B_a \rightarrow w \notin p)$

Notice that (9) is a logical consequence of the truth conditions and presupposition of (7) and that (9) represents the truth conditions of (10). Thus we explain the inference from (1)a to (1)b. A similar proposal can be found in Heim (2000).

(8) $\neg \forall w(w \in B_a \rightarrow w \in p)$
$\forall w(w \in B_a \rightarrow w \in p) \vee \forall w(w \in B_a \rightarrow w \notin p)$

(9) $\therefore \forall w(w \in B_a \rightarrow w \notin p)$

(10) *a believes that not-p*

Bartsch assumes that this is a *pragmatic* presupposition because it is easily canceled. Consequently, she suggests the presupposition arises as a result of "pragmatic application conditions".[2] Horn (1978) rightly takes Bartsch to task for failing to address why some predicates allow NR and others do not. It is unclear why these unspecified pragmatic application conditions should be in effect for *think* but not for *say*.

[1] In this context, this sentence is most naturally pronounced with stress on negation.
[2] Horn's (1978) translation of Bartsch's *pragmatische Verwendungsbedingungen*.

In fact, which predicates exhibit NR does not appear to be entirely predictable, as one would expect if the inference resulted from the application of a general pragmatic principle. For example in English, *want* is clearly NR, but *desire* is not. Furthermore, there is cross-linguistic variation in the class of NRPs. English *hope* is NR; German *hoffen* is not. Horn 1989 reports that Latin *sperare* was, but that French *espérer*, at least for some speakers, is not. For this reason, even the most successful approach to delineating the class of NRPs (Horn's 1975 mid-scalar generalization and its refinements, Horn 1989) must recognize that there are "semantically unmotivated lexical exceptions" (Horn 1989).

(11) A list of Neg-Raising predicates, arranged by semantic field (Horn 1989):
a. *think, believe, suppose, imagine, expect, reckon, feel*
b. *seem, appear, look like, sound like, feel like*
c. *be probable, be likely, figure to*
d. *want, intend, choose, plan*
e. *be supposed to, ought, should, be desirable, advise, suggest*

Given this mixed evidence, Horn & Bayer (1984) and Horn (1989) settle on an analysis in terms of what they call "short-circuited implicature," (SCI) an implicature that is in principle calculable but in fact a conventional property of some construction. The other examples of SCI that they offer are indirect speech acts, such as *Can you pass the salt?* and *Break a leg!* (see Sadock 1972, Searle 1975 and Morgan 1978). In the case of NRPs, the SCI they posit is equivalent to Bartsch's excluded middle presupposition. In this way, Horn & Bayer (1984) and Horn (1989) reconcile the defeasibility of NR with the (partial) arbitrariness in its application.

Each of these approaches has features to recommend it. I will argue for the semantic/pragmatic approach to Neg-Raising in this paper. I will, however, suggest a way of reconciling the defeasibility and conventionalization of NR different from that of Horn. Rather than grouping NR as an SCI with indirect speech acts like *Can you pass the salt?*, I group it with soft presuppositions in the sense of Abusch (2005). This will bring us closer in spirit to Bartsch (1973) than to Horn (1989), since we will view the excluded middle assumption associated with NRPs as a presupposition. I show that such an approach to NR has significant advantages in predictions about intricate patterns of NPI-licensing data.

1.2 Negative Polarity correlation with Neg-Raising

The waters muddy very quickly when one tries to resolve the grammatical status of NR based solely on intuitions about the implicational relationships between sentences like those in (1) and (2). Fortunately, there are other grammatical phenomena that correlate with our intuitions about these implications. The most trustworthy of these is the licensing of certain Negative Polarity Items (NPIs, though see Horn 1978 p.136 ff.). Lakoff (1969) (crediting Kajita) notes that certain "strict" NPIs, such as punctual *until,* additive *either*[3] and *in*+ indefinite time expression, are licensed by negation across an embedding predicate only when that predicate is NR. (For arguments that punctual *until* is a distinct lexical item from durative *until* see Karttunen 1974b, Declerck 1995, de Swart 1996, Giannakidou 2002.)

(12) Punctual *until*
a. *Mary left until yesterday
b. Mary didn't leave until yesterday

(13) *In* (+indefinite time expression) (cf. Hoeksema 1996)
a. *Bill has left the country in (at least two) years
b. Bill hasn't left the country in (at least two) years

(14) *Non-NR predicates*[4]
a. *Bill didn't claim that Mary would arrive until tomorrow
b. *Mary didn't claim that Bill had left the country in years

(15) *NR Predicates*
a. Bill doesn't think Mary will leave until tomorrow
b. Mary doesn't believe Bill has left the country in years

Note that not all NPIs display this pattern. The prototypical *any*/*ever*-type NPI, for example, is perfectly satisfied with a licenser separated from it by a non-NR predicate.

(16) a. Bill didn't claim that Mary had ever left the country.
b. Mary didn't claim that Bill had seen anything unusual.

(17) a. Bill didn't think that Mary had ever left the country.
b. Mary didn't believe that Bill had seen anything unusual.

Though many mysteries persist, the theory of NPI-licensing is quite advanced and has great predictive power. One motivation for this paper is to apply the results of the study of NPI-licensing to an intricate pattern of licensing involving NRPs first observed in Horn (1971). The hope is that the more solid ground of NPI-licensing theory will provide footing for attacking the fundamental questions about NR predicates.

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[3] I believe that *either* is a sound diagnostic for Neg-Raising, but involves complications that would take us too far afield.

[4] Horn (1978) notes several cases of strict NPIs licensed across non-NR predicates. He suggests that what distinguishes these cases is that they are formulations that conventionally convey a negative proposition.
(i) ?I don't know that I can trust you until you take a lie-detector test.
(ii) ?Mary didn't claim that anyone has been in the mine in years.
These are not great, but not as bad as expected. I have nothing of use to say about such cases.

1.3 Goals and Claims

This paper has two main goals. The first is to defend an approach to NR based on Bartsch (1973). I suggest that Bartsch (1973) is essentially correct but needs elaboration in terms of Abusch's (2005) theory of soft presuppositions. I argue for this approach mainly on the basis of its superiority over alternatives in accounting for NPI-licensing in NR environments. The second goal is to simultaneously defend a particular approach to the licensing of strict NPIs, which have traditionally served as a diagnostic for NR. The idea I will defend is that strict NPIs are licensed in Anti-Additive environments, following Zwarts (1998). Crucial support for these two views comes from the way they interact. In particular, we will see that projection of the excluded middle presupposition in certain embedded environments has a surprising effect on the licensing of strict NPIs.

My basic assumptions about Neg-Raising and NPI-licensing are laid out in Section 2. Detailed arguments in favor of these views, based on their interactions in presupposition projection environments, are given in Section 3. Section 4 concludes. In an Appendix I give further justification for certain assumptions I make about NPI-licensing.

2. Background

In this section, I lay out the assumptions I am making about the treatment of NRPs and the licensing of Negative Polarity Items (NPIs). In Section 2.1, I specify the account of NR that I favor. Essentially, I follow Bartsch's (1973) account, updating some details with recent work on soft presupposition triggers. In Section 2.2, I outline the theory of NPI-licensing that I assume. At the heart of the theory is the Fauconnier/Ladusaw Hypothesis, supplemented with Zwarts's work on degrees of negative strength (Zwarts 1998) and the logical properties of complex environments – the Monotonicity Calculus of Zwarts (1996). Finally in Section 2.3, I introduce and criticize an approach to strict NPI-licensing based on a syntactic analysis of NR.

2.1 Neg-Raising

In this paper, I adopt a version of Bartsch's approach to NR. As discussed in Section 1.1, Bartsch suggests associating NRPs with an excluded middle (EM) presupposition (18)b. Horn (1978) criticizes this account on the grounds that Bartsch's pragmatic application conditions for the presence of EM apply indiscriminately to all propositional attitudes. Given this, Bartsch incorrectly predicts that there are no non-NR attitude predicates.

(18) *a believes that p*
a. Truth conditions: $\forall w(w \in B_a \rightarrow w \in p)$
b. Presupposition: $\forall w(w \in B_a \rightarrow w \in p) \vee \forall w(w \in B_a \rightarrow w \notin p)$

One response to this criticism would be to say that the EM presupposition is lexically specified. The obvious difficulty with this response is that it apparently makes the presupposition "semantic". Bartsch (1973) specifically rules this out because she believes semantic presuppositions to be uncancelable. And yet, as we noted in the introduction, we are forced into some kind of lexical stipulation by semantically unmotivated lexical exceptions to the best available generalization about the class of NR predicates (Horn 1989). I would like to suggest that the EM presupposition is stipulated as a kind of 'soft' presupposition – in a sense to be clarified below. The value of this suggestion will come in our detailed analysis of NPI-licensing.

In section 2.1.1, we consider the evidence that the EM Bartsch associates with NRPs is a presupposition. In section 2.1.2, I sketch Abusch's 2005 account of soft presupposition triggers and suggest that NRPs fit naturally into this class. Finally, in section 2.1.3, I indicate how the non-NR reading of NRPs is captured.

2.1.1 Projection tests

A reliable test for identifying presuppositions is projection from embedded environments. When NR predicates are embedded in questions, in the antecedents of conditionals and under epistemic modals the effects are not obvious. For sake of comparison, I include the uncontroversially presuppositional, factive verb *regret*.

(19) a. Perhaps, John thinks Mary has left
b. If John thinks Mary has left, he'll do something impertinent
c. Does John think Mary has left?

(20) a. Perhaps, Mary said that Bill left
b. If Mary said that Bill left, she'll do something impertinent
c. Did Mary say that Bill left?

(21) a. Perhaps, Mary regrets that she said that
b. If Mary regrets that she said that, she'll do something impertinent
c. Did Mary regret that she said that?

We expect a presupposition of opinionatedness to project in (19)a-c, that John has a definite opinion about Mary's leaving. By contrast we expect no such 'said-p or said-not-p' presupposition in (20)a-c. Intuitions do not overwhelmingly confirm such a contrast. The fact that this presupposition, if there is any, is so weak, need not rule out a theory based on presupposition. It is well known that presupposition triggers differ in their strength, that is, in the extent to which it is possible to defeat the presupposition. Abusch (2005) puts forward an interesting hypothesis about the distinction between so-called soft and hard presupposition triggers, to which we turn now.

2.1.2 Hard vs. soft presupposition triggers

Presupposition triggers differ greatly in the ease with which their presuppositions may be canceled. Abusch (2005) divides triggers into two categories and labels them soft and hard triggers. The presuppositions of soft triggers are easily canceled by context; those of hard triggers are not. This distinction subsumes Karttunen's (1969) distinction between factives and semi-factives.

(22) a. I discovered that Fred left town.
b. I am angry that Fred left town.
c. Fred left town

(23) a. If tomorrow I discover that I told a lie today, I'll tell you.
b. If tomorrow I am angry that I told a lie today, I'll tell you.

Both (22)a and (22)b appear to presuppose (22)c. However, while (23)a may be uttered by someone who does not know whether they told a lie, (23)b cannot. This indicates that *discover* is a soft trigger, and *angry* a hard trigger. A similar distinction is discussed in Chierchia & McConnell Ginet (1990).[5]

In addition, Abusch lists among the soft triggers aspectual verbs such as *stop* and *start.* Similar data have recently been discussed in Simons 2001:

(24) If you have stopped smoking in the past year, you are eligible for a tax break.

Abusch suggests that soft triggers do not carry semantic presuppositions *per se*. Abusch thinks of semantic presuppositions as definedness conditions on context change potentials (Heim 1983). She suggests, by contrast, that soft triggers invoke alternatives, as a matter of convention. For example, the factive *know* invokes the alternative *be unaware*, and *stop* invokes the alternative *continue*. Though these alternatives seem natural, they must be lexically stipulated. Otherwise, as Abusch observes there is no way to distinguish *know* from the similar *be right*.[6] The invocation of the set of alternatives, then, triggers the application of a pragmatic principle that introduces the presupposition that one of the alternatives is true.

(25) Mary knows that Bill left.

(26) *a knows p = p & x believes p*
a is unaware that p = p & ~x believes p

(27) Alternatives:
{Bill left & Mary believes Bill left, Bill left & ~Mary believes Bill left}

(28) Presupposition:
(Bill left & Mary believes Bill left) OR (Bill left & ~Mary believes Bill left)
≡ Bill left

[5] A reviewer points out that there is much inter-speaker variation involving 'soft' triggers, see Karttunen (1973).
[6] *Know* and *be right* are both soft triggers and have the same components of meaning but *know* presupposes the truth of its complement, whereas *be right* asserts it.

The pragmatic principle at stake is an enrichment principle along the lines of Levinson's I-principle or Horn's R-principle. The specific formulation she gives is the following:

(29) *Generalization L*: If a sentence ψ is uttered in a context with common ground c and ψ embeds a clause φ which contributes an alternative set *Q*, then typically c is such that the corresponding local context d for φ entails that some element of *Q* is true.

(Abusch 2005)

The local contexts referred to are the derived environments created by context change potentials. The intention of this generalization is to mimic the projection behavior of semantic presuppositions. Crucial for us is the idea that, even though presuppositions can be generated in different ways and differ in cancelability, they project in the same way in embedded environments.

I suggest that we view NRPs as soft presupposition triggers.[7] It is natural to view NRPs as introducing a set of alternatives. In essence, this is the intuition underlying Bartsch's approach to NR. The alternative invoked by a NRP is its internal negation. The alternative to *believe* would be *doubt*, the alternative to *want to,* perhaps *want not to*.[8]

(30) *a believes p*
{ *a believes p*, *a believes ~p*}

(31) *a believes p* ∨ *a believes ~p*

Bringing NRPs under the umbrella of Abusch's soft triggers has the following advantages. The genesis of the pragmatic presupposition under Abusch's approach is ultimately due to Levinson's I-principle. This brings her theory into contact with Horn's approach to NR, which is based on his closely related R-principle. So, our approach is indeed very similar in spirit to Horn's (1989), see also Horn (2000). The main difference is in letter: we adopt Abusch's approach to the projection of soft presuppositions for our

[7] A reviewer suggests that this perspective suffers from the following problem. The mere prior assertion of the EM should suffice to trigger NR and therefore license strict NPIs in non-NR environments, contrary to fact.

(i) I have very strong feelings about his play. *I don't hope he goes until July.

I do not think this is a problem. The principles of NPI-licensing I endorse (2.2 below) pay attention only to the conventional properties of the NPI's environment and would not permit a contingent feature of context to license a strict NPI.

Notice that this commits me to representing cancellation with an overt operator like Beaver and Krahmer's 2001 Floating A, since it cannot be a contingent feature of context that prevents NPIs from being licensed in the non-NR reading of an NRP:

(ii) *I DON'T think Bill has visited in years.

[8] A lexical alternative might be *be loath to*, although as a reviewer points out this belongs to a different register from *want*.

EM presupposition. This will be important, for example, when embedding NRPs under belief and desire predicates.

2.1.3 A note on the non-NR reading

As mentioned in the Section 1, NRPs appear to be ambiguous. In certain contexts, a high negation is not understood as if in the lower clause. Under the syntactic approach, this is accounted for in terms of the position of negation at the level of interpretation. Under a Bartschian approach, on the other hand, the non-NR reading arises when the EM presupposition is canceled. I assume that the presupposition is canceled (or the Abusch-alternatives neutralized) by a syntactically present operator, like Beaver and Krahmer's (2001) Floating A. This operator cancels the presupposition and also affects the pronunciation of the statement (I repeat the cancellation case (4)b below, caps indicating stress):

(32) Bill DOESN'T think that Brutus killed Caesar.
LF: [Bill [**not** [**A** [thinks that Brutus killed Caesar.]]]

2.2 Negative Polarity

In this section, I lay out my assumptions about NPI-licensing. In section 2.2.1, I establish the format of my licensing principles. I follow Zwarts (1996), among others, in thinking of licensing in terms of environments, as opposed to c-commanding licensers. In section 2.2.2, I use Zwarts's (1998) notion of degrees of negative strength to formulate a licensing principle for strict NPIs. In particular, I claim that strict NPIs must be in Anti-Additive environments. In section 2.2.3, I apply these ideas to NRPs and show that their negations create AA environments and, therefore, license strict NPIs.

2.2.1 Licensing conditions on NPIs

The starting point for my approach to NPI-licensing is the familiar Fauconnier/Ladusaw Hypothesis (FLH). According to FLH, the licensing of NPIs depends on the logical properties of the environment in which an NPI occurs. Ladusaw (1979) identified the valid inference from sets to subsets (Downward Entailingness (DE-ness)) as a property necessary for licensing NPIs. (In the definitions below, I use '⇒' to stand for cross-categorial entailment).

(33) An NPI is licensed only if it occurs in the scope of an expression that denotes a Downward Entailing function.

(34) A function F is Downward Entailing iff for all A, B in the domain of F such that $A \Rightarrow B$, $F(B) \Rightarrow F(A)$.

I will depart slightly from this common statement of the licensing conditions on NPIs. Rather than requiring NPIs to appear in the scope of an expression that denotes a DE function, I require that an NPI occur in an environment that supports downward entailing inferences. Such a condition allows for a combination of expressions that do not themselves have a logical property to create an environment that does. Furthermore, some subconstituents of the scope of an expression that denotes a DE function might fail to support downward inferences if the environment contains another expression that interferes with downward inferences. Crucial use of such principles of licensing has been made by Heim (1984), Zwarts (1996) and Heim (2006) a.o. Our use of this principle mimics, in particular, Zwarts's (1996) Monotonicity Calculus.

(35) An NPI α is licensed in a sentence S only if there is a constituent β of S containing α such that β is Downward Entailing with respect to the position of α.

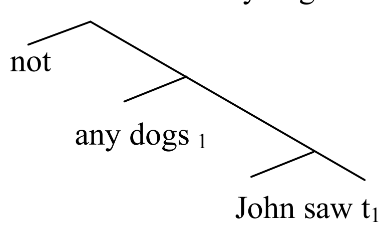
(36) A constituent β is Downward Entailing with respect to the position of α iff the function $\lambda x.\ [\![\ \beta[\alpha/v_{\sigma,i}\ ]\ ]\!]^{g[x\backslash<\sigma,i>]}$ is Downward Entailing (where $[\![\alpha]\!] \in D_\sigma$)

(37) $\beta[\alpha/\gamma]$ is the result of replacing α with γ in β

The definition in (36) simply says that a constituent is DE with respect to one of its subconstituents, if when you replace that subconstituent with a variable of the same type and abstract over it, the resulting function is DE.

Here is an example. The occurrence of *any* in (38) is licensed because the entire sentence is DE with respect to the position of *any*. This is so, because the function in (39) is DE, as demonstrated by the inference in (41).

(38) John didn't see any dogs



(39) $\lambda x_{<et,ett>}.[\![\text{not } [v_{<et,ett,2>} \text{ dog}]\ 1 \text{ John saw } t_1\ ]\!]^{g[x\backslash<et,ett,2>]}$

(40) $[\![$ two $]\!] \Rightarrow [\![$ any $]\!]$ (assuming $[\![$ any $]\!] = [\![$ some $]\!]$)

(41) John didn't see any dogs $\Rightarrow$ John didn't see two dogs

In the remainder of this paper, I will use a slightly more complex statement of such environment-related licensing principles. While the formulation is more complex it will ultimately make checking for NPI-licensing simpler. The idea is simply this: in checking whether the environment that an NPI occurs in is DE we do not need to pay attention to every expression that c-commands the NPI. Specifically, we can ignore any expression that c-commands the NPI but is taken by the NPI as an argument or is taken as an argument by the function that is the result of applying the NPI to another argument or ...

etc. Simply put, expressions whose denotations are arguments of the function denoted by an NPI do not affect the logical properties of the environment in which the NPI occurs.

To achieve this simplification we must first define an auxiliary notion, F(unction)-projection.

(42) F(unction)-projection

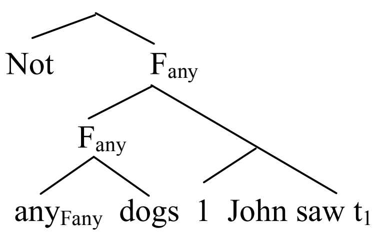
a. Every terminal node is an F-projection of itself.

b. If C is a branching node with daughters A, B, then C is an F-projection of A iff $[\![C]\!] = [\![A]\!]([\![B]\!])$ or B is a binding index.

c. F-projection* is the transitive closure of the F-projection relation

For example, the F-projections* of *any* are marked F_{any} in (43).

(43)



Now let's formulate a new principle for the licensing of NPIs based on this notion.

(44) An NPI α is licensed in a sentence S only if there is a constituent β containing α such that β is Downward Entailing with respect to the maximal F-projection* of α

Using this principle, the function that we have to check for DE-ness is much simpler: since the complement of *not* is the maximal F-projection* of *any*, the function to be checked for DE-ness is $\lambda p.[\![\text{ not } v_{<t,1>}]\!]^{g[p\backslash<t,1>]}$, which is just $[\![\text{ not }]\!]$.

2.2.2 Strict NPIs

Now let's use these notions to formulate the licensing principle for the strict NPIs introduced in Section 1.2. As mentioned above, an environment is DE if it licenses inferences from sets to subsets. For example,

(45) a. Not a single student read any books

b. Not every student read any books

(46) $[\![$ long book $]\!] \Rightarrow [\![$ book $]\!]$

(47) a. Not a single student read a book $\Rightarrow$ Not a single student read a long book

b. Not every student read a book $\Rightarrow$ Not every student read a long book

The valid inferences in (47), show that *not a single student* (the negation of an existential) and *not every student* (the negation of a universal) create DE contexts. This explains why *any* is licensed in (45). The general lesson to be taken away from these examples is summarized schematically below.

(48) The environments NOT(SOME(_)) and NOT(EVERY(_)) are both DE

Zwarts (1998) observes that DE-ness is not always sufficient to license an NPI (see also van der Wouden 1997). Some NPIs require environments that have logical properties in addition to DE-ness. Zwarts offers a classification of negative strength that is based on a generalization of
De Morgan's Laws:

(49) De Morgan's Laws
a. $\neg(X \wedge Y) \Leftrightarrow \neg X \vee \neg Y$
b. $\neg(X \vee Y) \Leftrightarrow \neg X \wedge \neg Y$

These equivalences can be split up into four entailment relations and generalized so that functions other than negation can be tested to see which parts of DeMorgan's Laws they validate.

(50) Strengths of Negation (Zwarts 1998)

(i)	$f(X) \vee f(Y) \Rightarrow f(X \wedge Y)$	Downward Entailing
(ii)	$f(X \vee Y) \Rightarrow f(X) \wedge f(Y)$	Anti-Additive
(iii)	$f(X) \wedge f(Y) \Rightarrow f(X \vee Y)$	Antimorphic
(iv)	$f(X \wedge Y) \Rightarrow f(X) \vee f(Y)$	

DE functions validate at least (50)i and (50)ii. An Anti-Additive function is one that in addition satisfies (50)iii. An Antimorphic function validates all four entailments in (50). Essentially, only sentential negation qualifies as Antimorphic. More natural language expressions satisfy the criteria for being Anti-Additive. For example, *not a single student* (= *no student*) creates an Anti-Additive environment; but *not every student* does not:

(51) a. Not a single student smokes and Not a single student drinks ⇒
Not a single student smokes or drinks
b. Not every student smokes and Not every student drinks =/⇒
Not every student smokes or drinks

So, from this we learn the following:

(52) NOT(SOME(_)) is ANTI-ADDITIVE
(53) NOT(EVERY(_)) is not ANTI-ADDITIVE

Zwarts's (1998) insight is that strict NPIs, which we introduced in Section 1.2 as a diagnostic for Neg-Raising, are sensitive to just this difference in negative strength.[9] They are not licensed by merely DE contexts, but instead require a context that is Anti-Additive.

(54) a. Not a single student has visited in years.
b. *Not every student has visited in years.

Given this, we can now formulate licensing principles for strict NPIs, requiring them to occur in Anti-Additive (henceforth, AA) environments.

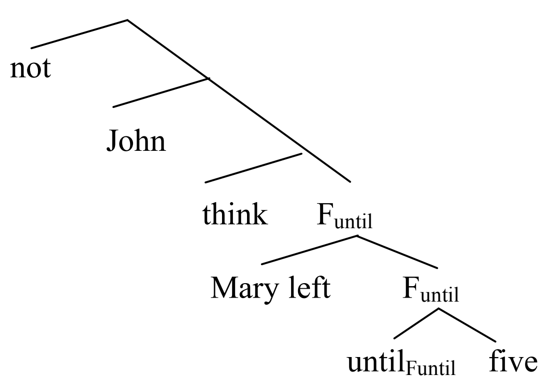
(55) A strict NPI α is licensed in a sentence S if there is a constituent β containing α such that β is Anti-Additive with respect to the maximal F-projection* of α

(56) A constituent β is Anti-Additive with respect to the position of α iff the function $\lambda x.\ [\![\ \beta[\alpha/v_{\sigma,i}\ ]\ ]\!]^{g[x \backslash < \sigma,i>]}$ is Anti-Additive (where $[\![\alpha]\!] \in D_\sigma$)

We need now to understand how this observation relates to the behavior of strict NPIs under NRPs. The principle (44) allows for the licensing of NPIs in the complements of both negated NR predicates and negated non-NR predicates. This follows since the combination of negation and a universal quantifier (over worlds) creates a DE environment. Our negated NR predicates are stronger than negated universals since, inferentially, they behave as if negation were below the NR predicate. That is, the negation of a NRP behaves like EVERY(NOT(_)). EVERY(NOT(_)) is Anti-Additive – in contrast to NOT(EVERY(_)). So, I hypothesize that strict NPIs are allowed under negated NR predicates because these are in fact Anti-Additive (henceforth, AA) environments.

Consider the NRP example (57) – the F-projections* of *until* are marked F_{until}.

(57) John doesn't think Mary left until five.



[9] I will stick to the traditional term 'strict NPI' for those under discussion, as opposed to Zwarts's term 'strong'. The reason for this is that Zwarts and others associate the term 'strong' with minimizers, which I believe have a broader distribution.

We need to make sure that our semantics for NRPs makes (57) AA with respect to the embedded clause (the maximal F-projection* of *until*). This is demonstrated in the next section.

2.2.3 Negated NRPs create AA environments

Van der Wouden (1995) observes that (in Dutch) negated NRPs show (some of) the licensing capabilities of AA functions. In other words, the negation of a NRP licenses NPIs like the negation of an existential.

(58) a. Bill doesn't think Sue has visited in years.
b. *Bill doesn't know that Sue has visited in years.

Van der Wouden stops short of giving a semantics for NRPs. The challenge is to give a semantics that is universal but whose negation acts like the negation of an existential. The presuppositional account of NR reconciles the universal semantics of NRPs with the AA-ity of their negations. Let's see how.

Recall that I assume that NRPs have lexical entries of the form (59), where M is NRP's modal base.

(59) For any proposition P, and individual x,
$[\![\text{NRP}]\!](P)(x)$
(i) presupposition: $M(x) \subseteq P$ or $M(x) \cap P = \varnothing$
(ii) truth condition: $M(x) \subseteq P$

The crucial part of our story is what happens when you negate an NRP that carries an excluded middle (EM) presupposition:

(60) $[\![\text{not}]\!]([\![\text{NRP}]\!](P)(x))$
(i) presupposition: $M(x) \subseteq P$ or $M(x) \cap P = \varnothing$
(ii) truth condition: $M(x) \not\subseteq P$

Under what conditions is (60) defined? The negation of a sentence inherits the presuppositions of that sentence unmodified. So, (60) is defined in the same cases as (59). The assertion of (60) is simply the negation of the universal assertion of (59).

Notice that the presupposition and assertion of (60) come together to entail the second disjunct of the presupposition:

$$\text{(61)}\quad \left.\begin{array}{l} \text{((60)i)}\ M(x) \subseteq P \ \text{ or } M(x) \cap P = \varnothing \\[1em] \text{((60)ii)}\ M(x) \not\subseteq P \end{array}\right\} \Longrightarrow \quad M(x) \cap P = \varnothing$$

So let's see now what we predict about Anti-Additivity: is the entailment in (62) predicted to be valid by our semantics?

(62) not NRP(P)(x) and not NRP(Q)(x) $\Rightarrow$ not NRP(P$\vee$Q)(x)

(63) a. [[not NRP(P)(x)]]
(i) presupposes: $M(x) \subseteq P$ or $M(x) \cap P = \varnothing$
(ii) asserts: $M(x) \not\subseteq P$
(iii) Together (i) and (ii) entail: $\boxed{M(x) \cap P = \varnothing}$
b. [[not NRP(Q)(x)]]
(i) presupposes: $M(x) \subseteq Q$ or $M(x) \cap Q = \varnothing$
(ii) asserts: $M(x) \not\subseteq Q$
(iii) Together (i) and (ii) entail: $\boxed{M(x) \cap Q = \varnothing}$

c. [[not NRP(P$\vee$Q)(x)]]
(i) presupposes: $M(x) \subseteq P\vee Q$ or $M(x) \cap P\vee Q = \varnothing$
(ii) asserts: $M(x) \not\subseteq P\vee Q$

If (63)a and (63)b are both true, then the presupposition of (63)c is satisfied. The reason is that (63)a entails that no M-world is a P-world and (63)b entails that no M-world is a Q-world, thus no M-world is a P$\vee$Q-world. The assertions of (63)a and (63)b entail that M is non-empty, it follows from this and the fact that no M world is a P$\vee$Q world that the assertion of (63)c is true. So for arbitrary P, Q, the truth of (63)a and (63)b guarantees the truth of (63)c. Thus, negated NRPs do create an AA context.

Note in this regard that the inference in (64) is intuitively valid. Compare the invalidity of (65).[10]

(64) John doesn't think Mary left and John doesn't think Bill left. $\Rightarrow$
John doesn't think Mary left or Bill left

(65) John isn't certain that Mary left and John isn't certain that Bill left. $\Rightarrow$
John isn't certain that Mary left or Bill left.

So, now we have a sound account of the licensing of strict NPIs under negated NRPs. Furthermore, it can also be shown that this approach is superior to an alternative

[10] A reviewer wonders if this test isn't circular. We test for NR using strict NPIs. We test whether NRPs license strict NPIs with inferences like (64). The validity of (64) appears to depend on a specified NR reading. I do not think this is a problem. NPI-licensing is not our only test for NR. There is an intuition based on entailments. Furthermore, the non-NR reading of such sentences, on which (64) is invalid, must generally be marked with stress on negation or on the predicate (cf. Horn 1989 p. 315). In fact, most naïve native speakers require a bit of convincing that the non-NR reading exists. To my ear, the contrast between (64) and (65) is clear enough to distinguish their licensing abilities.

approach based on a syntactic rule of Neg-Raising. We now turn to showing this in Section 2.3.

2.3 Critique of syntactic licensing

One alternative to the AA approach to licensing strict NPIs is to assume a licensing condition that dovetails with a syntactic approach to NR. A hypothesis as to why these NPIs interact with NR in this way is already present in Lakoff (1969). In fact, this interaction is pointed to as an argument in favor of the syntactic theory of NR. Lakoff proposes that strict NPIs are required to be clausemates with negation. Under the syntactic theory, this immediately accounts for the contrast between the licensing abilities of non-NR predicates and of NRPs (cf. (14) and (15)). A negation occurring above a NRP can have been base-generated in the complement clause, as a clausemate with the strict NPI. A negation above a non-NR predicate cannot have such a source.

(66) Neg-Raising predicate
    Interpretive level: [John thinks [Mary not left until Friday]])
    Surface: [John does not think [Mary ~~not~~ left until Friday]]

In the above derivation, *until Friday* and *not* are clausemates at the level of interpretation though on the surface they are separated by an intervening predicate *think*. If negation is a clausemate of *until* under a non-NR predicate, the two remain clausemates on the surface:

(67) non Neg-Raising predicate
    Interpretive level: [John claims [Mary not left until Friday]])
    Surface: [John claims [Mary did not leave until Friday]]

In the next two subsections, I will argue against the clausemate condition as the appropriate licensing condition for strict NPIs. In particular, I argue that it is neither necessary nor sufficient for a strict NPI to be a clausemate with its licenser. What is crucial, I argue, is the semantic properties of the environment of the strict NPI. The crucial cases involve strict NPIs not licensed by a clausemate negation and strict NPIs licensed by a negative operator located outside of its clause.

2.3.1 Clausemate negation is not sufficient

We have already seen cases in which clausemate negation is not sufficient to license a strict NPI. Recall that the NPIs used to diagnose NR exhibit a need for strong negative contexts: punctual *until* and *in weeks* require an Anti-Additive context.

(68) a. *Not every student arrived until 5 o'clock.
    b. Not a single student arrived until 5 o'clock.
(69) a. *Not every student has visited Bill in (at least two) years.
    b. Not a single student has visited Bill in (at least two) years.

These examples pose a challenge to the clausemate condition on the licensing of *until/in years*. In (68)a and (69)a, the NPIs appear to be clausemates with negation but are not licensed.[11] If one suggests that these prenominal negations do not count as clausemates for the NPIs, then (68)b and (69)b become a mystery.

2.3.2 Clausemate negation is not necessary

In this section, we look at sentences in which a strict NPI is separated from negation by a non-NR predicate but still licensed.[12] We see that the AA hypothesis predicts that such cases are grammatical.

The crucial test case we will construct is one in which negation is separated from a strict NPI by a predicate that is an existential quantifier over worlds. Under the Anti-Additive hypothesis, we predict that the negation of an existential quantifier ought to license a strict NPI. On the other hand, a theory that relies on a clausemate condition for explaining the distribution of strict NPIs predicts that strict NPIs under negated existential predicates should be acceptable only if the predicate is NR. But, as Horn (1978) argues, no existential predicate is NR[13], cf. (70). Thus, the clausemate hypothesis predicts they should be ungrammatical.

(70) a. Bill is allowed to smoke and Bill is not allowed to smoke (Contradictory)
b. Bill is allowed to smoke and Bill is allowed not to smoke (Consistent)

As the sentences in (71) show, the prediction of the clausemate hypothesis is incorrect. The AA hypothesis does much better. Because *can* and *allow* are existential predicates, their negations should create AA contexts that license strict NPIs.

(71) a. An applicant is not allowed to have left the country in at least two years[14]

---

[11]An objection to the AA hypothesis is sometimes raised on the basis of examples such as (i), since *few* NP is not AA.

(i) a. ?Few students arrived until 5 o'clock.
b. ?Few students have visited Bill in weeks.

A reviewer also offers examples of *not many* licensing strong NPIs. I simply note these as a potential problem, discussion of which would take us too far afield. For a possible approach, see Gajewski to appear.

[12] The content of this section is inspired by Guerzoni's (2001) discussion of the licensing of n-words in Italian.

[13] Indeed, Horn (1989) argues that no Tolerant predicate is NR. See Löbner 1985 for discussion of Tolerance.

[14] A reviewer suggests that this example gets worse if we take out the *at least two*.
(i) ?An applicant is not allowed to have left the country in years

b. An applicant can't have left the country in at least two years.

An advocate of the clausemate hypothesis might object that these sentences involve non-finite clauses and that non-finite clauses do not count as boundaries for the clausemate condition. This is a reasonable objection. It does predict, however, that if we replace the existential predicates in (71) with universal predicates such as *require* and *have to*, the result should be grammatical. This is incorrect.

(72) a. *An applicant is not required to have left the country in at least two years
b. *An applicant doesn't have to have left the country in at least two years.

This is problematic for the clausemate hypothesis, but conforms to the AA hypothesis. As we know, NOT(EVERY(_)) is not an Anti-Additive environment. Thus, we correctly predict the ungrammaticality of (72). So, whether or not non-finite clauses count for assessing the clausemate condition, the AA hypothesis is more successful in predicting the licensing of strict NPIs.

Before drawing such a sanguine conclusion, one difficulty should be noted. In (72) and (71), I have used example in which negation and strict NPI are separated by a non-finite clause boundary. This is not an innocent oversight. Many researchers have identified finiteness as a relevant factor in determining whether a clause boundary interferes with NPI-licensing (see, a.o., Horn 1978, Giannakidou 1997). And it must be admitted that examples analogous to (71) involving finite clause boundaries are much degraded.

(73) *It is not certain that Bill has left the country in at least two years.
(74) ??It is not possible that Bill has left the country in at least two years.

So it appears that there may still be some room for a locality condition to apply in the licensing of strict NPIs.[15]

2.4 Summary of Section 2

In this section we have seen that the clausemate condition does not give an adequate account of the distribution of *until* and *in years*. On the one hand, being clausemates with negation is not sufficient:

(68)a *Not every student arrived until 5 o'clock.

On the other hand, it is not necessary:

I agree, but believe this has nothing to do with NPI-licensing. I think the vagueness of the bare plural conflicts slightly with the formality of the permissions associated with *allow*.

[15] Another factor to be considered is the mood of the embedded clause (again, see Horn 1978). Discussion of mood is beyond the scope of this paper.

(71)a An applicant is not allowed to have left the country in at least 2 years.

By contrast, we have seen that an approach to licensing these items based on Zwarts's (1998) classification of negative strength gives a natural account of the facts when combined with the presuppositional theory of NR.

The syntactic approach to NR, however, is still compatible with the negative strength approach to the distribution of *until*/*in years*. In the next section, we will see that when we consider a broader range of constructions, the syntactic theory faces a number of problems. The presuppositional theory, by contrast, extends naturally to cover the data.

APPENDIX to Section 2: *it's not true that*

It is well known that the negation of *it is true/the case that* does not license strict NPIs (see discussion in Horn 1989 p. 327). A reviewer suggests that this casts doubt on the hypothesis that strict NPIs are licensed in AA environments. If predicates such as *true* are truly redundant, then this argument is correct; the environment should have the same properties as sentential negation and, therefore, license strict NPIs.

(75) a. *It's not true that Bill has visited Mary in weeks.
b. *It's not the case that Bill arrived until yesterday.

In this appendix, I give a possible response. Assume that *true* is not simply redundant. It is often suggested that predicates such as *true* cancel the presuppositions of their complements (see work related to Bochvar 1939 and discussion in Horn 1989 p.126 ff.). I adopt this idea despite warnings in Atlas (1974), Horn (1989), Beaver and Krahmer (2001). Under this approach, the truth conditions of a *true* statement conjoin the truth conditions of the complement with the presuppositions of the complement. ['ϕ_α' stands for a sentence with truth condition ϕ and presupposition α.]

(76) TRUE(ϕ_α)
Truth conditions: $\phi \wedge \alpha$
Presupposition: T

Recall, now, how one tests whether an environment is AA. The crucial entailment is in (77), where F represents the function denoted by the environment (see Section 2.2.1).

(77) $F(A) \wedge F(B) \Rightarrow F(A \vee B)$

To see how the presupposition-canceling properties of *true* can affect assessment of AA-ity, we must be explicit about the presupposition-projection properties of disjunction. Here's a standard treatment along the lines of Karttunen and Peters 1979.[16]

[16] Karttunen and Peters (1979, Rule 11 p.50) rule (ϕ^e represents the entailments of ϕ, ϕ^i its conventional implicature):
$$[\phi \vee \chi]^e = \phi^e \vee \chi^e$$

(78) $\phi_{\alpha} \vee \chi_{\beta}$

Truth conditions: $\phi \vee \chi$

Presupposition: $(\phi \vee \beta) \wedge (\chi \vee \alpha)$

Combining the above analysis of *true* and the meaning rule for disjunction in (78) yields invalidity for the inference in (79).

(79) $\sim\text{TRUE}(\phi_{\alpha}) \wedge \sim\text{TRUE}(\chi_{\beta}) \Rightarrow \sim\text{TRUE}(\phi_{\alpha} \vee \chi_{\beta})$ INVALID

It is easy to see why, when we unpack the contribution of the TRUEs.

(80) $\sim(\phi \wedge \alpha) \wedge \sim(\chi \wedge \beta) \Rightarrow \sim((\phi \vee \chi) \wedge (\phi \vee \beta) \wedge (\chi \vee \alpha))$ INVALID

The premise is true and the conclusion false when ϕ and χ are both true and α and β both false. It is worth noting that the implication is valid in the other direction:

(81) $\sim((\phi \vee \chi) \wedge (\phi \vee \beta) \wedge (\chi \vee \alpha)) \Rightarrow \sim(\phi \wedge \alpha) \wedge \sim(\chi \wedge \beta)$

So, the environment is DE and, therefore, predicted to license weak NPIs. This is also correct.

(82) It's not true that Bill said anything intelligent.

I leave it to the reader to the see that paying such attention to the presupposition projection properties of disjunction does not affect the cases already discussed.

I am unable to provide a natural, intuitively invalid instance of (79). So, at it stands, this explanation is technical and incomplete. I do think, however, that it is worth observing that the invalidity of (79) is a consequence of plausible views about the semantics of *true/case* and the presupposition-projection properties of disjunction.

3. Presupposition Projection and Neg-Raising

In this section, we extend our account of NR and strict NPI-licensing to cases that involve non-trivial principles of presupposition projection. It is well known that strict NPIs can be licensed under NRPs also when the negation above the NR predicate is part of a more complex construction, cf. Horn (1978). In this section, we will see how the predictions of our account about these cases depend on how the excluded middle (EM) presupposition projects through the constructions. In section 3.1, we look at the case of NRPs in the scope of negative quantifiers. In section 3.2, we look at the particularly interesting case of NRPs embedded under other NRPs. In section 3.3, we offer an explanation of a puzzling asymmetry revealed in the discussion in section 3.2.

$[\phi \vee \chi]^{i} = (\phi^{e} \vee \chi^{i}) \wedge (\phi^{i} \vee \chi^{e})$

3.1 Negative quantifiers

Consider the sentence (83) in which the subject of the NR predicate is a negative existential:

(83) No one thought Bill would leave until tomorrow.

(84) Every one thought Bill wouldn't leave until tomorrow.

Here the negative subject 'triggers' NR. This is indicated by the fact that (83) may be understood as conveying (84) and that punctual *until* is licensed in the embedded clause. This is expected under the approach to NR we are pursuing in this paper. As I will now demonstrate, the context in which *until* occurs, namely the complement of *think*, is Anti-Additive.

Consider the representation below in which branching nodes are annotated with their presuppositions. (O stands for ⟦*one*⟧ – the set of people.)

(85) ⟦ think ⟧(p)(x)

Truth condition: $B_x \subseteq \{w: p(w)=1\}$

Presupposition: $B_x \subseteq \{w: p(w)=1\} \vee B_x \subseteq \{w: p(w)\neq 1\}$

(86) $O \subseteq \{x: B_x \subseteq \{w: p(w)=1\} \vee B_x \subseteq \{w: p(w)\neq 1\}\}$

No one_1 $\quad$ $B_x \subseteq \{w: p(w)=1\} \vee B_x \subseteq \{w: p(w)\neq 1\}$

x_1

thinks $\quad$ that p

I assume with Heim (1983) (and against Beaver 1994) that the presuppositions of quantificational structures are universal. In other words, I am claiming that the sentence *no one thinks that p* presupposes that everyone either thinks that p or thinks that not p – in other words, everyone has an opinion about p. More formally, this gives us (87) as the presupposition of *no one thinks that p*, while its truth conditions are in (88).

(87) $O \subseteq \{x: B_x \subseteq \{w: p(w)=1\} \vee B_x \subseteq \{w: p(w)\neq 1\}\}$

(88) $O \cap \{x: B_x \subseteq \{w: p(w)=1\}\} = \varnothing$

Together (87) and (88) entail (89). If everyone has an opinion about p and no one holds the belief that p is true, then everyone must think p is false.

(89) $O \subseteq \{x: B_x \subseteq \{w: p(w)\neq 1\}\}$

("Everyone thinks that not-p")

Given, this it should be clear that *no one thinks that p and no one thinks that q* is predicted to entail that *no one thinks that p or q*. If every person's belief worlds are worlds in which p is not true and every person's belief worlds are worlds in which q is not true than every person's belief worlds are worlds in which $p \vee q$ is not true. This satisfies the presupposition of *no one thinks that p or q* and affirms its truth.

(90) No one thinks Bill is here and no one thinks Sue is here ⇒
No one thinks that Bill is here or Sue is here
(i.e., No one thinks that there's one of them here)

Note that without the EM presupposition the context is not AA.

Also notice that to explain the licensing of *until* in (83), a syntactic account would have to decompose the negative subject into negation and a universal quantifier:

(91) a. SS: [$\underbrace{\text{every one not}}_{\text{“no one”}}$ thought [that Bill _ left]]
b. LF: [every one _ thought [that Bill not left]]

As Horn (1978) has already pointed out, this is problematic. While decomposition of negative quantifiers like *no one* is often proposed, most evidence supports decomposing it into negation and an existential/indefinite, cf. Kratzer (1995), Potts (2000), Penka & von Stechow (2001). Having two such different decompositions of a single form should be avoided.

As we move forward it will be useful to keep in mind the following results from this section (such results about the logical properties of complex constructions are nicely outlined in Zwarts 1996):

(92) a. EVERY(EVERY(NOT(_))) is Anti-Additive
b. EVERY(NOT(EVERY(_))) is not Anti-Additive

Now that we have seen the role that presupposition projection plays in licensing NPIs, we will see how asymmetries in presupposition projection account for asymmetries in the licensing of NPIs. In so doing, we will build on argument of Horn's (1971) against the syntactic account of NR.

3.2. (Partial) Cyclicity

In his classic paper on NR, Fillmore 1963 supports his syntactic analysis of NR by pointing out that NR operates cyclically. That is, if a negation appears at the top of an uninterrupted sequence of NRPs, the negation can be understood as if it took scope beneath the lowest of the NRPs. (*Imagine, think* and *want* are all NR predicates.)

(93) a. I don't imagine Mary thinks Fred wants to leave.[17]
b. I imagine Mary thinks Fred wants not to leave.

(94) [I _ imagine [Bill thinks [Mary wants [Fred not to go]]]]

This is a *prima facie* compelling argument. Horn 1971 (reporting joint work with J. Morgan) observes, however, that this cyclicity does not hold as generally as Fillmore had thought. In particular while the sequence of a NR belief-predicate embedding a NR desire-predicate permits cyclic NR, the reverse sequence of a NR desire predicate embedding a NR belief predicate does not. For example, (95)a implies (95)b, but (96)a does not imply (96)b.

(95) a. I don't believe Bill wanted Harry to die.
b. I believe Bill wanted Harry not to die.

(96) a. I don't want Bill to believe Harry died
b. I want Bill to believe Harry didn't die.

Horn and Morgan support this subtle judgment with sturdier judgments concerning the licensing of strict NPIs:

(97) a. I don't believe John wanted Harry to die until tomorrow
b. *I don't want John to believe Harry died until yesterday
(based on Horn 1971 (4')[18])

The phenomenon is not limited to these two predicates, but extends to other doxastic and deontic/bouletic predicates more generally:

(98) a. Mary doesn't think Bill should have left until yesterday
b. *Mary shouldn't think Bill left until yesterday
(cp. Mary should think Bill didn't leave until yesterday)

(99) a. Bill doesn't imagine Sue ought to have left until yesterday
b. *Bill ought not imagine Sue left until yesterday.
(cp. Bill ought to imagine Sue didn't leave until yesterday.)

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[17] In the interest of full disclosure I note that Fillmore's original example violates the observation made by Horn and Morgan reported below, since *want* embeds *think*:

(i) I don't believe that he wants me to think that he did it.
(Fillmore 1963, p.220)

[18] I have changed the original examples slightly to control for the scope of *until*. Wherever possible, I choose a complement for *until* that precludes its being construed with a higher clause.

According to Horn (1971)/(1978), who is following Lindholm 1969, the contrast in (97) is related to *believe* having two distinct senses. One more semantically bleached, parenthetical sense permits NR, the other more semantically contentful sense does not. Horn proposes that the NR sense of *believe* is not available in the complement of *want*. I will pursue a different analysis of these facts. I suggest that our presuppositional view of NR combined with our view of NPI-licensing yields an elegant explanation of this pattern.

3.3 Explaining the contrast in (97)

In this section, I show that the presuppositional approach to NR offers an explanation of the contrast in (97). To see how, we need to take a brief detour into the presupposition-projection properties of sentence-embedding predicates, such as *believe* and *want*.

It is well known that desire predicates differ from belief predicates in their presupposition-projection properties (cf. Karttunen 1974, Heim 1992). A belief predicate, on the one hand, asserts that its complement is a belief of its subject and presupposes that the presuppositions of its complement are beliefs of its subject, as well. A desire predicate, on the other hand, asserts that its complement is a desire of its subject but presupposes that the presuppositions of its complement are *beliefs* of its subject. For example, (101) presupposes that Bill believes he has a cello and (102) presupposes not that Bill wants to have a cello, but that he believes he has one.[19]

(100) Bill will sell his cello.
Presupposition: Bill has a cello.

(101) Bill thinks he will sell his cello.
Presuppositions: Bill thinks he has a cello.

[19] A reviewer questions this account, pointing out that one can utter (i) without presupposing that you believe there is (or will be) a King of France.

(i) I want to be the King of France.

This is correct, however I believe it is related to another well-known property of desire predicates. The presuppositions of their complements can be satisfied by entailments of previously expressed desires.

(ii) I want France to be a monarchy. I want to meet its King.

In other words, I believe (i) can be felicitous so long as it is common ground that the speaker wants there to be a King of France. We do not need to change our semantics to account for this. I suggest following Roberts (1996), who follows Heim (1992), in analyzing (ii) and closely related cases of modal subordination in terms of local accommodation into the doxastic modal base.

(102) Bill wants to sell his cello.

Presupposition: Bill thinks he has a cello

(Not: Bill wants to have a cello)

3.3.1 think > want

Let's now consider how the presuppositional analysis captures the cyclicity of NR in a sentence like (97)a. For ease of exposition, I will assume a presupposition-projection mechanism along the lines of Karttunen & Peters (1979). See, for example, Karttunen & Peters Rule 4 (p. 49). (The use of this rule is purely expository, see Appendix Two for a calculation of the presuppositions in a system where presuppositions are modeled as definedness conditions) In K&P's system constituents denote an ordered pair: the first coordinate is its extension and the second coordinate is its conventional implicature (which we will refer to as a presupposition). When Functional Application applies to two ordered pairs the first coordinate of the output is the extension of the function applied to the extension of the argument. The second coordinate of the output is the second coordinate of the function applied to the *extension* of the argument conjoined with the output of applying the heritage function to the extension of the function and presupposition of the argument. The heritage function determines what becomes of the argument's presupposition given what function applies to the argument. When the function is an attitude verb the output of the heritage function is the statement that the presupposition of the argument is believed by the attitude holder. Crucial for us is (104), which says that the presuppositions of the complement of an attitude verb project as beliefs of the subject of the attitude.

(103) <u>K&P Function Application</u>

$<\alpha, \beta>(<\gamma, \delta>) \;= <\alpha(\gamma), \beta(\gamma) \wedge h(\alpha,\delta)>$

(104) When α is an attitude predicate,

$h(\alpha,\delta) = \lambda x.\ B_x \subseteq \{u: \delta(u)=1\}$

Given this perspective we may state the following lexical entries and heritage rules.

(105) $[\![$ think $]\!]^w$ ($B_{x,w}$ stands for the worlds compatible with x's beliefs in w)

truth condition: $\lambda p.\lambda x.B_{x,w} \subseteq \{w: p(w)=1\}$

presupposition: $\lambda p.\lambda x.[B_{x,w} \subseteq \{w: p(w)=1\} \vee B_{x,w} \subseteq \{w: p(w)\neq 1\}]$

heritage: $h([\![$ think $]\!]^w, dom(p)) = \lambda x.B_{x,w} \subseteq dom(p)$[20]

(106) $[\![$ want $]\!]^w$ ($D_{x,w}$ stands for the worlds compatible with x's desires in w)

truth condition: $\lambda p.\lambda x.D_{x,w} \subseteq \{w: p(w)=1\}$

presupposition: $\lambda p.\lambda x.[D_{x,w} \subseteq \{w: p(w)=1\} \vee D_{x,w} \subseteq \{w: p(w)\neq 1\}]$

heritage: $h([\![$ want $]\!]^w, dom(p)) = \lambda x.B_{x,w} \subseteq dom(p)$

[20] Here I use 'dom(p)' or 'domain of p' as shorthand for its presuppositional component.

Using these rules and definitions, we can calculate the truth conditions and presupposition of (107)a.

(107) a. John doesn't think Fred wants Mary to leave.

b.

not

$\alpha \left\{ \begin{array}{l} \text{TC: } B_{j,@} \subseteq \{w: D_{f,w} \subseteq \{v: p(v)=1\}\} \\ \text{P: (i) } B_{j,@} \subseteq \{w: D_{f,w} \subseteq \{v: p(v)=1\} \vee D_{f.w} \subseteq \{v: p(v) \neq 1\}\} \\ \quad \text{(ii) } B_{j,@} \subseteq \{w: D_{f,w} \subseteq \{v: p(v)=1\}\} \vee B_{j,@} \subseteq \{w: D_{f,w} \not\subseteq \{v: p(v)=1\}\} \end{array} \right.$

John believes

$\beta \left\{ \begin{array}{l} \text{TC: } \lambda w.\ D_{f,w} \subseteq \{v: p(v)=1\} \\ \text{P: } \lambda u. D_{f,u} \subseteq \{w: p(w)=1\} \vee D_{f.u} \subseteq \{w: p(w) \neq 1\} \end{array} \right.$

Fred wants Mary to leave (= p)

I have indicated the two coordinates of the semantic values of constituents α and β next to them in a bracket (TC indicates the truth conditions, P the presuppositions). The entire structure (107)b inherits the presuppositions of α. Presupposition (ii) of α is the EM presupposition associated with *believe*. Presupposition (i), on the other hand, derives from the application of the heritage function to the presupposition of β. Now the assertion of (107)a is (108).

(108) $B_{j,@} \not\subseteq \{w: D_{f,w} \subseteq \{v: p(v)=1\}\}$
("John DOESN'T think Fred wants Mary to leave"[21])

This combined with presupposition (ii) of α (107)b gives us (109).

(109) $B_{j,@} \subseteq \{w: D_{f,w} \not\subseteq \{v: p(v)=1\}\}$
("John thinks Fred DOESN'T want Mary to leave")

This combined with presupposition (i) of α in (107)b entails that

(110) $B_{j,@} \subseteq \{w: D_{f,w} \subseteq \{v: p(v) \neq 1\}\}$
("John think Fred wants Mary not to leave")

The fact that we can get the negation to "go all the way down" makes the context of the most deeply embedded clause AA. Why? Notice that (110) is of the form (EVERY(EVERY(NOT(_))). We have already seen that this is an AA context.

3.3.2 want > think

[21] I use caps here to indicate intonational prominence and to disambiguate in favor of a non-NR reading.

If we try to use this reasoning when the predicates are in the reverse order we run in to a problem. Consider again the case of (97)b, repeated as (111)a:

(111) a. John doesn't want Fred to think Mary left.

b. [not [α [John wants [β [Fred to think] [Mary left (= p)]]]]]

α { TC: $D_{j,@} \subseteq \{w: B_{f,w} \subseteq \{v: p(v)=1\}\}$
P: (i) $B_{j,@} \subseteq \{w: B_{f,w} \subseteq \{v: p(v)=1\} \vee B_{f.w} \subseteq \{v: p(v) \neq 1\}\}$
(ii) $D_{j,@} \subseteq \{w: B_{f,w} \subseteq \{v: p(v)=1\}\} \vee D_{j,@} \subseteq \{w: B_{f,w} \not\subseteq \{v: p(v)=1\}\}$

β { TC: $\lambda w.\ B_{f,w} \subseteq \{v: p(v)=1\}$
P: $\lambda u.B_{f,u} \subseteq \{w: p(w)=1\} \vee B_{f.u} \subseteq \{w: p(w) \neq 1\}$

The assertion of (111)a is (112).

(112) $D_{j,@} \not\subseteq \{w: B_{f,w} \subseteq \{v: p(v)=1\}\}$
("John DOESN'T want Fred to think Mary left")

This together with presupposition (ii) of α in (111) entails that

(113) $D_{j,@} \subseteq \{w: B_{f,w} \not\subseteq \{v: p(v)=1\}\}$
("John wants Fred NOT to think Mary left")

In the case of (107)a, we were able to use presupposition (i) of α to infer the final 'cyclic' step of NR. In this case we cannot.

(114) DOES NOT FOLLOW
$D_{j,@} \subseteq \{w: B_{f,w} \subseteq \{v: p(v) \neq 1\}\}$
("John wants Fred to think Mary didn't leave")

In other words, one can believe that Fred has an opinion whether Mary left, want that it not to be the case that he believes Mary left and still not want Bill to believe Mary didn't leave. To see that this environment is not AA, note that (113) is of the form EVERY(NOT(EVERY(_))), which we have already seen is not AA.

We have shown that (97)a is Anti-Additive with respect to the position of the most deeply embedded clause and that (97)b is not. Given that the complement of *want* is the maximal F-projection* of *until* in (97)a, we have an explanation for why *until* is licensed. Similarly, given that the complement of *think* is the maximal F-projection* of *until* in (97)b, we have an explanation for why it is not licensed.

3.4 Summary of Section 3

In this section, we have seen that the presuppositional theory of NR in conjunction with the negative strength approach to NPI-licensing extends naturally to an account of NR in negative quantificational structures and in cases of negated stacked NRPs. In particular, this theory predicts that NR is not always cyclic, as observed by Horn and Morgan. The syntactic theory by contrast faces obstacles of unnatural decomposition for negative quantifiers and overgeneration with stacked NR predicates.

4. Conclusion

This paper has explored the representation of Neg-Raising in the grammar and its consequences for the licensing of strict NPIs. We have argued that Neg-Raising is represented in the grammar as a (soft) presupposition and that strict NPIs are subject to licensing conditions in the spirit of the Fauconnier/Ladusaw Hypothesis. Our statement of the licensing conditions makes use of innovations contributed primarily by Zwarts (1996) and Zwarts (1998), which suggest the difference strengths of negation must be distinguished and that environments are what matter for licensing (not necessarily c-commanding licensers). Our argument is based on the advantages of empirical coverage of NPI-licensing facts related to NR environments. We have shown that a puzzling asymmetry in strict NPI-licensing under stacked NRPs receives a natural explanation under the perspective of this paper.

APPENDIX ONE: Further issues in the licensing of strict NPIs

In the main body of this paper, we have endorsed a theory like Zwarts (1998) in which a certain class of NPIs requires licensing by a logical property stronger than DE-ness, in particular Anti-Additivity. In this section, we address how such a theory interacts with von Fintel's (1999) recent proposal for amending the Fauconnier/Ladusaw Hypothesis (FLH). I conclude that the licensing principles for strict NPIs must be stated in terms of AA-ity defined with standard entailment, not von Fintel's Strawson entailment. One problematic case remains: superlatives. A brief examination of Romance n-words suggests that this problematic case is not an idiosyncrasy of English.

A.1 Strawson Entailment

There is a class of environments in which *any* and *ever* are licensed even though the environments do not appear to license DE inferences, for example in the scope of [*only* DP].

(115) Only Bill ever talked to anyone.

(116) a. Only Bill ate a vegetable
b. #Therefore, only Bill ate kale

Intuitively, the inference in (116) fails because (116)a does not tell us which vegetable Bill ate – if it wasn't kale, then (116)b is not true. Von Fintel (1999) suggests weakening the notion of DE-ness relevant to licensing *any* and *ever*. He points out that while (116) is not valid, a related inference is valid namely (116) under the assumption that all the sentences involved in the inference are defined. This analysis depends on a presuppositional analysis of *only* along the lines of Horn (1969). The definition of Strawson DE-ness is in (119).

(117) Given an individual a and set P
$[\![\text{only}]\!](a)(P)$ is defined only if $a \in P$
When defined $[\![\text{only}]\!](a)(P) = 1$ iff there is no $y \neq a$ such that $y \in P$

(118) Cross-Categorial Entailment
a. For p, q of type t: $p \Rightarrow q$ iff p = False or q = True.
b. For f, g of type $<\sigma,\tau>$: $f \Rightarrow g$ iff for all x of type σ: $f(x) \Rightarrow g(x)$.

(119) Strawson Downward Entailingness
A function f of type $<\sigma,\tau>$ is Strawson-DE iff for all x, y of type σ such that $x \Rightarrow y$ and f(x) is defined: $f(y) \Rightarrow f(x)$.

Amending FLH so that *any/ever* need only appear in the scope of a Strawson DE function allows for NPI-licensing in (115), since (120) is valid.

(120) Bill ate kale.
Only Bill ate a vegetable.
Therefore, Only Bill ate kale.

From von Fintel's (1999) approach we can extract a notion of Strawson Entailment apart from DE-ness (see Herdan and Sharvit 2006):

(121) Strawson Entailment ($\Rightarrow_S$)
a. For p, q of type t: $p \Rightarrow_S q$ iff p = False or q = True.
b. For f, g of type $<\sigma,\tau>$: $f \Rightarrow_S g$ iff for all x of type σ such that f(x) and g(x) are defined: $f(x) \Rightarrow_S g(x)$.

A.2 Strawson Anti-Additivity

Given this, one might (and should) ask whether we want to replace every mention of entailment in the theory of NPI-licensing with Strawson Entailment. In particular, we should ask if this affects the licensing conditions of strict NPIs. Should the statement of the licensing principle refer to Strawson Anti-Additivity?

(122) A function F is Strawson Anti-Additive iff $F(A) \wedge F(B) \Leftrightarrow_S F(A \vee B)$[22]

Apparently not, since under this definition *only* DP comes out Strawson Anti-Additive[23]

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[22] $A \Leftrightarrow_S B$ iff $A \Rightarrow_S B$ and $B \Rightarrow_S A$
[23] A fact also noted in Rullmann 2003.

and *only* DP fails to license strict NPIs (the failure of *only* to license such NPIs was noted by Atlas 1996):[24]

(123) a. *Only John arrived until 5 o'clock.
b. *Only John visited Marry in years.
c. (Only John likes pancakes.) *Only John likes waffles either.

[*Only* DP] uncontroversially validates the Left-to-Right implication:

(124) Only John drinks and Only John smokes
$\therefore$Only John drinks or smokes

The Right-to-Left Implication is Strawson valid:

(125) John drinks and John smokes
Only John drinks or smokes
$\therefore$Only John drinks and only John drinks

So, from the perspective of Strawson Entailment, [*only* DP] is Anti-Additive but does not license strict NPIs. I argue that it is standard entailment and not Strawson Entailment that figures in the statement of the licensing condition on strict NPIs. By way of supporting this generalization, I observe that two other environments that von Fintel identifies as Strawson DE fail to license strict NPIs. Note that these constructions also validate the Left-to-Right implication of (122).

(126) Adversatives (see also Giannakidou 2006)
*Sue is sorry that Bill arrived until five

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[24] The attentive reader may notice that *only* does appear to license strict NPIs in one context: when it functions adjectivally.

(i) This is the only tapir I have seen in weeks.

This does not tell us anything about adverbial *only*. Rather, it is another piece of evidence that adjectival *only* is a distinct lexical item and, in fact, a superlative. Bhatt 2002 and Herdan 2005 give semantic arguments that adjectival *only* is a superlative. It should be noted that in many languages adverbial *only* cannot function as an adjective, e.g., German, in which *einzig* replaces *nur* in adjectival contexts. Furthermore, in some dialects of English, adjectival *only* is overtly superlative, pronounced *onliest* (see Montogomery & Hall 2004 for Smoky Mountain English). Hoeksema 1986 also argues that adjectival *only* is superlative, citing Dutch *de enigste*.

A reviewer raises the following problem for equating adjectival *only* with superlatives:
(ii) a. The only thing you need worry about is money
b. *The most important thing you need worry about is money

*Sue is sorry that Bill has visited John in years

(127) Antecedent of a Conditional
*If Bill arrived until five, Mary was upset.
*If Sue has visited Bill in years, then Mary is upset.

(128) Bill is sorry Sue is here and Bill is sorry Fred is here ⇒
Bill is sorry Sue is here or Fred is here

(129) If Bill arrived at five, then Mary is upset and
if Sue arrived at six, then Mary is upset ⇒
If Bill arrived at five or Sue arrived at six, then Mary is upset

So, these constructions would also count as Anti-Additive, if our underlying notion of entailment were Strawsonian. Let's refer to such functions as Strawson Anti-Additive (SAA) and to functions that are AA on the standard notion of entailment as standard AA.

Note that there is one construction analyzed by von Fintel as (merely) Strawson Downward Entailing that defies this trend. This is the case of superlatives, which von Fintel (1999) assigns the semantics in (130).

(130) $[\![$ the...–est$]\!](P)(Q)(\alpha)$ is defined only if $Q(\alpha) = \text{True}$
If defined, $[\![$the...–est$]\!](P)(Q)(\alpha)=\text{True}$ iff $(\forall x: x\neq\alpha \ \&\ Q(x) = \text{True})\ \iota dP(x)(d)< \iota dP(\alpha)(d)$

Under this analysis, superlatives turn out to be Strawson-DE. Furthermore, superlatives do intuitively validate the Left-to-Right direction of (122):

(131) Erin is the tallest girl in this class and Erin is the tallest girl in that class ⇒
Erin is the tallest girl in this class or that class.

So, superlatives do create SAA environments. And, actually, in this case, strict NPIs are licensed in relative clauses in the scope of a superlative morpheme.[25]

(132) Superlatives
a. Erin is the tallest girl John has seen in years.
b. The tallest girl John had seen until Friday walked in the room.

If we wish to maintain that the licensing of strict NPIs requires a standard AA environment and not merely a SAA environment, then we need a different semantics for superlatives: one in which superlatives create a strict-AA environment.

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[25] Incidentally, the acceptability of strict NPIs in superlatives argues against a dependence of strict NPIs on negative morphology, proposed as a licensing condition by van der Wouden 1997 and Horn 1996 (fn. 12).

At this time, I do not have a well-motivated analysis of this kind. In the next section, we see, however, that the exception of superlatives can also be found in the licensing of Romance n-words.

A.3 English strict NPIs and n-words in Italian and Spanish

A good deal of research has been done on the distribution of Romance n-words. Much of what has been discovered about their distribution overlaps with the distribution of strict NPIs in English. For example, it has been frequently proposed that Romance n-words require strong negative licensers, e.g., Anti-Additive operators (cf. Ladusaw 1992, Guerzoni 2001, a.o.). Consider (134)a, in which an n-word fails to be licensed by the merely DE *meno di tre studenti* 'fewer than three students.'

(133) Anti-Additive
Nessuno ha visto niente
N-one has seen n-thing
"No one saw anything."

(134) a. Non-Anti-Additive
*Meno di tre studenti hanno mangiato niente — Italian
Fewer than three students have eaten n-thing

b. Conditional
??Se Maria accorgesse niente, sarebbe un problema — Italian
If Mary noticed n-thing, it-would-be a problem

c. *Only*
??Solo Maria ha visto nessuno degli studenti — Italian
Only Mary has seen n-one of-the students

d. Adversatives — Italian
*Mi spiacerebbe che tu vedessi nessuno
I would be sorry that you saw n-one

(Alonso-Ovalle and Guerzoni 2004)

It has also been noted that this generalization is not adequate. Romance n-words are not licensed by SAA environments, such as the antecedents of conditionals, the scope of [*only* DP], and adversatives, cf. (134). This has led to a variety of proposals for accounting for the distribution, e.g., replacing Downward Entailingness with Non-Veridicality in the licensing conditions on NPIs (Giannakidou 1997) or deriving their distribution from a conventional implicature (Alonso-Ovalle and Guerzoni 2004).

What I would like to point out is merely that superlatives also license the existential concord reading of n-words in Italian and Spanish:

(135) ´E l'idea pi`u stupida che abbia mai avuto nessuno — Italian

it's the-idea more stupid than has ever had n-one
'It's the stupidest idea anyone ever had' (Acquaviva 1997)

(136) Es la ultima vez que te digo nada Spanish
it's the last time that you I-tell n-thing
'It's the last time that I tell you anything' (Herburger 1997)

This parallels closely the distribution of strict NPIs in English and sharpens the puzzle about the NPI-licensing status of superlatives.

Summary of Appendix One

In this section we asked whether the change to FLH suggested by von Fintel (1999) should be extended to all NPI-licensing statements. Specifically, we asked whether Anti-Additivity should be replaced with Strawson Anti-Additivity in the licensing principles for strict NPIs. The answer was no. Merely Strawson Anti-Additive operators, such as [*only* DP], do not license strict NPIs. One problematic case was noted. Superlatives appear to be merely Strawson AA, but license strict NPIs. Furthermore we saw that superlatives' patterning with strict-AA licensers extends to the case of Negative Concord licensing in Spanish and Italian – suggesting that the pattern in English is not a fluke.

APPENDIX TWO: Deriving the asymmetry in (97)

In this appendix, I give a more detailed calculation of the meanings of (97)a and (97)b. I then show that the stated claims about Anti-Additivity hold. In this version, I model presuppositions as definedness conditions.

(97) a. I don't believe John wanted Harry to die until tomorrow
b. *I don't want John to believe Harry died until yesterday

Implementing this account formally encounters one technical difficulty. That difficulty is how to analyze the contribution of presuppositional constituents contained in other presuppositional constituents. This analysis requires that the presuppositions of the embedded predicates do not contribute to the meaning of the Excluded Middle presupposition of the predicates that embed them. More specifically, I propose that the presuppositions of the embedded item are "canceled" within the Excluded Middle presupposition. They do contribute to the presuppositions of the larger constituent through projection. Let's see what I mean by this by spelling out some concrete lexical entries ['$B_{w,x}(u)$' abbreviates 'u is compatible with x's beliefs in w']:

(1) $[\![\text{believe}]\!]^{w}(p)(x)$ is defined only if
(i) $\forall u\, [\, B_{w,x}(u) \rightarrow [\, p(u) = 1 \text{ or } p(u) = 0 \,]\,]$
(projection of the presuppositions of the embedded clause)
(ii) $\forall u\, [\, B_{w,x}(u) \rightarrow p(u) = 1 \,]$ or $\forall u\, [B_{w,x}(u) \rightarrow p(u) \neq 1 \,]$
when defined, $[\![\text{believe}]\!]^{w}(p)(x) = 1$ iff $\forall u\, [\, B_{w,x}(u) \rightarrow p(u) = 1 \,]$

In this definition, the crucial part is the consequent of the second disjunct in presupposition (ii). Here we have crucially written '$p(u)\neq 1$' rather than '$p(u)=0$'. This effectively cancels the presupposition of p within the Excluded Middle presupposition of $[\![\text{believe}]\!]$, think of this as 'external' negation. In this particular lexical entry, the distinction does not ultimately make a difference for the definedness conditions. Clause (i), which projects the presuppositions of the embedded clause, guarantees that p is true or false in each of the subjects' belief worlds. So, if every belief world w is such that p is not true in w, then every belief world w is such that p is false in w. That is, we might just as well have written '$p(u)=0$'.

This decision to use '$p(u)\neq 1$' rather than '$p(u)=0$' in the Excluded Middle presupposition has a more dramatic effect in the lexical entry of *want*. The reason being, of course, that the projection clause of the definedness conditions does not match up with the Excluded Middle presupposition as it did in the lexical entry of *believe*. ['$D_{w,x}(u)$' abbreviates 'u is compatible with x's desires in w']

(2) $[\![\text{ want }]\!]^w(p)(x)$ is defined only if

(i) $\forall u\,[\, B_{w,x}(u) \rightarrow [\, p(u) = 1 \text{ or } p(u) = 0 \,]\,]$

(projection of the presuppositions of the embedded clause)

(ii) $\forall u\,[\, D_{w,x}(u) \rightarrow p(u) = 1 \,]$ or $\forall u\,[\, D_{w,x}(u) \rightarrow p(u) \neq 1 \,]$

when defined, $[\![\text{want }]\!]^w(p)(x) = 1$ iff $\forall u\,[\, D_{w,x}(u) \rightarrow p(u) = 1 \,]$

Clause (i) of the definedness conditions projects the presuppositions of the embedded clause, requiring that the subject of *want* believe them. In the proofs below, I make tacit use of the assumption that the modal bases are not empty. For reasons of space, these proofs are greatly abbreviated and not all steps are justified.

(3) $[\![\text{want}]\!]^w(\lambda u.[\![\text{believe}]\!]^u(p)(a))(c)$ is defined only if

(i) $\forall u\,[\, B_{w,c}(u) \rightarrow [\, [\![\text{believe}]\!]^u(p)(a) = 1 \text{ or } [\![\text{believe}]\!]^u(p)(a) = 0 \,]]$ iff

$\forall u[B_{w,c}(u) \rightarrow [\forall v[B_{u,a}(v) \rightarrow p(v) = 1\,] \vee \forall v[B_{u,a}(v) \rightarrow p(v)=0]]]$

(ii) $\forall u[D_{w,x}(u)\rightarrow [\![\text{believe}]\!]^u(p)(a)=1]$ or $\forall u[D_{w,x}(u)\rightarrow [\![\text{believe}]\!]^u(p)(a) \neq 1]$ iff

$\forall u[D_{w,c}(u) \rightarrow \forall v[B_{u,a}(v) \rightarrow p(v) = 1]]$ or $\forall u[D_{w,c}(u) \rightarrow \neg\forall v[B_{u,a}(v)\rightarrow p(v) = 1]]$

when defined, $[\![\text{want}]\!]^w(\lambda u.[\![\text{believe}]\!]^u(p)(a))(c) = 1$ iff

$\forall u\,[\, D_{w,c}(u) \rightarrow [\![\text{believe}]\!]^u(p)(a) = 1\,]$ iff

$\forall u[\, D_{w,c}(u) \rightarrow \forall v[B_{u,a}(v) \rightarrow p(v) = 1]\,]$

(4) Equivalences used in (3)

a. $[\![\text{believe}]\!]^u(p)(a) = 1$ iff $\forall v[B_{u,a}(v) \rightarrow p(v) = 1]$

b. $[\![\text{believe}]\!]^u(p)(a) = 0$ iff $\forall v[B_{u,a}(v) \rightarrow p(v) = 0]$

This is precisely the result we want. The negation of (3) does not entail that c wants a to believe that not p. It merely entails that c wants a to not believe that p. It furthermore presupposes that c believes that a either believes that p or that not p. However, without any further postulates about the relationship of belief worlds to desire worlds, this does not entail that c wants a to believe that not p. It may be that practically we do assume beliefs constrain desire in this way. It is my hypothesis that this constraint is not imposed by the grammar.

Furthermore, given these proofs, it is simple to show that (5) does not contain a constituent that is Anti-Additive with respect to the most deeply embedded clause. Thus we correctly predict that strict NPIs are not licensed in (5).

(5) $[\![\text{c doesn't want a to believe that p}]\!]^w = 1$ iff
$[\![\text{want}]\!]^w(\lambda u.[\![\text{believe}]\!]^u(p)(a))(c) = 0$
(negation preserves presuppositions)

Now we show that (6)ii does not follow from (6)i.

(6) i. $[\![\text{want}]\!]^w(\lambda u.[\![\text{believe}]\!]^u(p)(a))(c) = 0$ and $[\![\text{want}]\!]^w(\lambda u.[\![\text{believe}]\!]^u(q)(a))(c) = 0$
ii. $[\![\text{want}]\!]^w(\lambda u.[\![\text{believe}]\!]^u(p\vee q)(a))(c) = 0$

To do so, we construct a simple model in which (i) holds and (ii) does not.

(7) a. D_c: $w \mapsto \{w1, w2\}$
b. B_a: $w1 \mapsto \{w3, w4\}$
$w2 \mapsto \{w5, w6\}$
c. $p(w3) = p(w5) = 1$; $p(w4) = p(w6) = 0$
d. $q(w3) = q(w5) = 0$; $q(w4) = q(w6) = 1$
e. $\forall u\in\{w3, w4, w5, w6\}\ p\vee q(u)=1$

In every one of c's desire worlds there is a belief world of a in which p is false. Similarly for q. A quick inspection of (3) shows that this verifies (i). But (ii) does not hold. In fact, $[\![\text{want}]\!]^w(\lambda u.[\![\text{believe}]\!]^u(p\vee q)(a))(c) = 1$ in this model. In every one of c's desire worlds, $p\vee q$ is true in every one of a's belief worlds. This makes $[\![\text{want}]\!]^w(\lambda u.[\![\text{believe}]\!]^u(p\vee q)(a))(c)$ both defined and true.

Thus the environment $[\![\text{want}]\!]^w(\lambda u.[\![\text{believe}]\!]^u(\_\ )(a))(c)$ is not Anti-Additive since it fails the inference in (8).

(8) $F(A) \wedge F(B) \Rightarrow F(A\vee B)$

This contrasts with the case in which *want* is embedded under *believe*. In that case, the inference in (8) does indeed go through.

(9) $[\![\text{believe}]\!]^w(\lambda u.[\![\text{want}]\!]^u(p)(a))(c)$ is defined only if
(i) $\forall u\ [\ B_{w,c}(u) \rightarrow [\ [\![\text{want}]\!]^u(p)(a) = 1$ or $[\![\text{want}]\!]^u(p)(a) = 0\ ]\ ]$ iff
$\forall u[B_{w,c}(u) \rightarrow [\ [\forall v[D_{u,a}(v) \rightarrow p(v) = 1\ ] \vee \forall v[D_{u,a}(v) \rightarrow p(v)\neq 1]\ ] \wedge$
$\forall v[B_{u,a}(v) \rightarrow [p(v) = 1 \vee p(v) = 0\ ]\ ]]]$
(ii) $\forall u[B_{w,c}(u)\rightarrow [\![\text{want}]\!]^u(p)(a)=1] \vee \forall u[B_{w,c}(u)\rightarrow [\![\text{want}]\!]^u(p)(a)\neq 1]$ iff
$\forall u[B_{w,c}(u) \rightarrow [\ \forall v[D_{u,a}(v) \rightarrow p(v) = 1] \wedge \forall v[B_{u,a}(v) \rightarrow [p(v)=1$ or $p(v)=0]]]]$

or

$\forall u[B_{w,c}(u) \rightarrow [\exists v[B_{w,a}(v) \wedge p(v)\neq 1 \wedge p(v)\neq 0] \vee \exists v[D_{w,a}(v) \wedge p(v)\neq 1]]]$

when defined, $[\![\text{believe}]\!]^{w}(\lambda u.[\![\text{want}]\!]^{u}(p)(a))(c) = 1$ iff

$\forall u\ [\ B_{w,c}(u) \rightarrow [\![\text{want}]\!]^{u}(p)(a) = 1\ ]$ iff

$\forall u[B_{w,c}(u) \rightarrow [\forall v[D_{u,a}(v) \rightarrow p(v) = 1] \wedge\ \ \forall v[B_{u,a}(v) \rightarrow [p(v){=}1 \text{ or } p(v){=}0]]]]$

(10) Equivalences used in (9)

$[\![\text{want}]\!]^{u}(p)(a) = 1$ iff $\forall v[D_{u,a}(v) \rightarrow p(v) = 1] \wedge \forall v[B_{u,a}(v) \rightarrow [p(v){=}1 \text{ or } p(v){=}0]]$

$[\![\text{want}]\!]^{u}(p)(a) \neq 1$ iff $\exists v[B_{u,a}(v) \wedge p(v)\neq 1 \wedge p(v)\neq 0] \vee \exists v[D_{u,a}(v) \wedge p(v)\neq 1]$

$[\![\text{want}]\!]^{u}(p)(a) = 0$ iff $\forall v[D_{u,a}(v) \rightarrow p(v)\neq 1] \wedge \forall v[B_{u,a}(v) \rightarrow [p(v){=}1 \text{ or } p(v){=}0]]$

(11) $[\![\text{believe}]\!]^{w}(\lambda u.[\![\text{want}]\!]^{u}(p)(a))(c) = 0$ iff (i), (ii) and (iii)

(i) $\forall u[B_{w,c}(u)\rightarrow[[\forall v[D_{u,a}(v)\rightarrow p(v) = 1\ ] \vee \forall v[D_{u,a}(v)\rightarrow p(v)\neq 1] \wedge$
$\forall v[B_{u,a}(v) \rightarrow [p(v) = 1 \vee p(v) = 0\ ]]]]$

(ii) $\forall u[B_{w,c}(u) \rightarrow [\ \forall v[D_{u,a}(v) \rightarrow p(v) = 1] \wedge \forall v[B_{u,a}(v) \rightarrow [p(v){=}1 \text{ or } p(v){=}0]]]]$

or

$\forall u[B_{w,c}(u) \rightarrow [\exists v[B_{w,a}(v) \wedge p(v)\neq 1 \wedge p(v)\neq 0] \vee \exists v[D_{w,a}(v) \wedge p(v)\neq 1]]]$

(iii) $\neg\forall u[B_{w,c}(u) \rightarrow [\forall v[D_{u,a}(v) \rightarrow p(v) = 1] \wedge \forall v[B_{u,a}(v) \rightarrow [p(v){=}1 \text{ or } p(v){=}0]]]]$

We may simplify this as follows:

(12) (ii) and (iii) are equivalent to (iv):

(iv) $\forall u[B_{w,c}(u) \rightarrow [\neg\forall v[B_{w,a}(v) \rightarrow p(v){=}1 \vee p(v){=}0] \vee \neg\forall v[D_{w,a}(v) \rightarrow p(v){=}1]]]$

Notice now that (i) and (iv) entail (v):

(13) (v) $\forall u[B_{w,c}(u) \rightarrow [\forall v[D_{u,a}(v) \rightarrow p(v)\neq 1] \wedge \forall v[D_{u,a}(v) \rightarrow [p(v) = 1 \vee p(v) = 0]]]]$

Notice now that if (v) holds for another proposition q, then (13) holds of $p\vee q$ as well. Why? If in every one of c's belief worlds p is false in every one of a's desire worlds and the same holds of q, then in every one of c's belief worlds $p\vee q$ is false in every one of a's desire worlds. Furthermore if in every one of c's belief worlds, p is either true or false in each of a's belief worlds, and the same holds of q, then in every one of c's belief worlds, $p\vee q$ is true or false in each of a's belief worlds. These facts verify that $[\![\text{believe}]\!]^{w}(\lambda u.[\![\text{want}]\!]^{u}(p\vee q)(a))(c) = 0$. The inference in the other direction is straightforward. The crucial step in our reasoning, what differentiated this case from the last, was the use of presupposition (i) to draw the inference in (95).

Thus, the environment $[\![\text{believe}]\!]^{w}(\lambda u.[\![\text{want}]\!]^{u}(\_)(a))(c)$ is Anti-Additive. In this Appendix, we have seen how Zwarts's (1998) approach to the distribution of strict NPIs, a Barstch/Heim approach to Neg-Raising, and some independently justified principles of presupposition projection dovetail neatly and predict an intricate contrast in the licensing of strict NPIs under multiple NRPs.

References

Abusch, D. (2005). Triggering from Alternative Sets and Projection of Pragmatic Presuppositions. Ms. Cornell University

Acquaviva, P. (1997). The Logical Form of Negation: A Study of Operator-Variable Structures in Syntax. (New York:Garland.)

Alonso-Ovalle, L. & Guerzoni, E. (2004). Double negatives, negative concord and metalinguistic negation. (In M. Andronis, E. Debenport, A. Pycha & K. Yoshimura (eds.), Proceedings of CLS 38-1:Main Session (pp.15-31) Chicago: CLS Publications.)

Atlas, J. (1974). Presupposition, ambiguity, and generality: a coda to the Russell-Strawson debate on referring. Unpublished Ms. Pomona College.

Atlas, J. (1996). 'Only' noun phrases, pseudo-negative generalized quantifiers, negative polarity items, and monotonicity. Journal of Semantics, 13, 265-328.

Bartsch, R. (1973). “Negative Transportation” gibt es nicht. *Linguistische Berichte* 27.

Beaver, D. (1994). When variables don't vary enough. (In M. Harvey & L.Santelmann (eds.), Proceedings from Semantics and Linguistic Theory IV (pp.35-60) Ithaca, NY:CLC Publications.)

Beaver, D. & E. Krahmer (2001) Presupposition and Partiality: Back to the Future. Journal of Logic, Language and Information 10(2), pp. 147-182

Bhatt, R. (2002). The raising analysis of relative clauses: Evidence from adjectival Modification. Natural Language Semantics, 10, 43-90.

Bochvar, D., (1939), Ob odnom trehznachom iscislenii i ego primeneii k analizu paradoksov klassicskogo rassirennogo funkcional ‘nogo iscislenija’. In: *Matematiciskij sbornik*, **4** (English translation (1981): “On a Three-valued Logical Calculus and Its Applications to the Analysis of the Paradoxes of the Classical Extended Functional Calculus.” In: *History and Philosophy of Logic* **2**:87-112)

Chierchia, G. & McConnell-Ginet, S. (1990). Meaning and Grammar an Introduction to Semantics. MIT Press.

Declerck, Renaat. 1995. The problem of not...until. Linguistics 33: 51-98.

Fillmore, C. (1963). The position of embedding transformations in grammar. Word, 19, 208–231.

von Fintel, K. (1993). Exceptive constructions. Natural Language Semantics, 1, 123-148.

von Fintel, K. (1997). Bare plurals, bare conditionals and ‘only’. Journal of Semantics, 14, 1–56.

von Fintel, K. (1999). NPI-licensing, strawson entailment and context dependency. Journal of Semantics, 16, 97–148.

Fodor, J. D. (1970). The linguistics description of opaque contexts. Ph.D. Dissertation, MIT.

Gajewski, J. (to appear). Licensing strong NPIs. (In the Proceedings of the Penn Linguistics Colloquium 31)

Giannakidou, A. (1997). The landscape of polarity items. Ph.D. Dissertation, University of Groningen.

Giannakidou, A. (2002). UNTIL, aspect and negation: a novel argument for two *until*s. (In B. Jackson (ed.), Proceedings from Semantics and Linguistic Theory XII (pp. 84-103) Ithaca, NY:CLC Publications.)
Giannakidou, A. (2006). Only, emotive factives, and the dual nature of polarity dependency. Language*,* 82: 575-603.
Guerzoni, E. (2001). Italian n-words as NPIs. Incontro di Grammatica Generativa.
Heim, I. (1983). On the Projection Problem for Presuppositions. (In M. Barlow, D. Flickinger & M. Westcoat (eds.), Proceedings of West Coast Conference on Formal Linguistics 23 (pp. 114-126) University of Stanford.)
Heim, I. (1984). A note on negative polarity and downward entailingness. (In C. Jones & P. Sells (eds.), Proceedings of the North East Linguistics Society 14 (pp. 98-107) GLSA.)
Heim, I. (1992). Presupposition projection and the semantics of attitude verbs. Journal of Semantics, 9, 183–221.
Heim, I. (2000). Degree operators and scope. (In B. Jackson and T. Matthews (eds.), Proceedings from Semantics and Linguistic Theory X (pp. 40-64) Ithaca, NY: CLC Publications.)
Heim, I. (2006). Remarks on comparative clauses as generalized quantifiers. Ms. MIT.
Herburger, E. (1997). The negative concord puzzle revisited. Natural Language Semantics, 9, 289-333.
Herdan, S. (2005). Who knows the only reading of *only*? Snippets, 10, 7-8.
Herdan, S. & Y. Sharvit (2006). Definite and Non-definite Superlatives and NPI-licensing. Syntax 9:1-31.
Hoeksema, J. (1986). Monotonie en superlatieven. (In C. Hoppenbrouwers, J. Houtman, I. Schuurman and F. Zwarts (eds.) Proeven van taalwetenschap, Nederlands Instituut RUG, Gronigen (pp. 38-49))
Hoeksema, J. (1996). In days, weeks, months, years, ages: A class of temporal negative polarity items. Ms. Groningen.
Horn, L. R. (1969). A presuppositional analysis of *only* and *even*. (In CLS 5 (pp. 97-108))
Horn, L. R. (1971). Negative transportation: Unsafe at any speed? (In CLS 7 (pp. 120–133))
Horn, L. R. (1975). Neg-raising predicates: toward an explanation. (In CLS 11 (pp. 279-294))
Horn, L. R. (1978). Remarks on neg-raising. (In P. Cole (ed.), Syntax and Semantics 9: Pragmatics (pp. 129–220) New York:Academic Press.)
Horn, L. R. (1984). Toward a new taxonomy for pragmatic inference: Q-based and R-based implicature. (In D. Schiffrin (ed.) Meaning, form and use in context: Linguistic applications (pp. 11–42) Georgetown University Press)
Horn, L. R. (1989). A natural history of negation. (Chicago: Univeristy of Chicago Press)
Horn, L. R. (2000). Conditional perfection as pragmatic strengethening. *Journal of Pragmatics* 32: 289-326.
Horn, L. R. & Bayer, S. (1984). Short-circuited implicature: A negative contribution. Linguistics and Philosophy, 7, 397-414.
Karttunen, L. (1969). Some observations on factivity. Papers in Linguistics, 5, 55-69.
Karttunen, L. (1973). STOP: is there a presupposition? Unpublished ms.

Karttunen, L. (1974a). Presupposition and linguistic context. Theoretical Linguistics, 1, 181–193.
Karttunen, L. (1974b). Until. In CLS 10. 284-297.
Kratzer, A. (1995). Stage-level and individual-level predicates. (In G. Carlson & F. J. Pelletier (eds.), The Generic Book (pp.125-175) Chicago: University of Chicago Press.)
Ladusaw, W. (1979). Polarity as inherent scope relations. Ph.D. Dissertation, University of Texas at Austin.
Ladusaw, W. (1992). Expressing negation. (In C. Barker & D. Dowty (eds.), Proceedings from Semantics and Linguistic Theory II (pp.237-259) Ithaca,NY:CLC Publications.)
Lakoff, R. (1969). A syntactic argument for negative transportation. In CLS 5 pp. 149–157.
Lindholm, J.M. (1969). Negative-Raising and Sentence Pronominalization. (In Chicago Linguistics Society 5, pp. 148-158.)
Löbner, S. (1987). Quantification as a Major Module of Natural Language Semantics. (In J. Groenendijk, D. de Jongh and M. Stokhof (eds.) Studies in Discourse Representation Theory and the Theory of Generalized Quantifiers. (pp.53-85) Foris, Dordrecht.)
Morgan, J. (1978). Two types of convention in indirect speech acts. (In P. Cole (ed.) Syntax and Semantics 9: Pragmatics (pp. 261–280) New York: Academic Press.)
Montgomery, M. B. & Hall, J. S. (2004). Dictionary of Smoky Mountain English. Universtiy of Tennessee Press.
Penka, D. & von Stechow, A. (2001). Negativen indefinita unter modalverben. (In Reimar Muller and Marga Reis (ed.) Modalitat und modalverben im deutschen, Sonderheft 9 edition.)
Potts, C. (2000). When even no's neg is splitsville. URL: http://ling.ucsc.edu/Jorge/potts.html.
Prince, E. (1976). The syntax and semantics of neg-raising, with evidence from French. Language, 52, 404–426.
Roberts, C. (1996). Anaphora in intensional contexts. In S. Lappin (ed.) (1996) The Handbook of Contemporary Semantic Theory, Basil Blackwell.
Ross, J. R. (1973). Slifting. (In M.P. Schutzenburger, M.Gross & M. Halle (eds.), The formal analysis of natural languages: Proceedings of the first international conference (pp. 133–169) Mouton: The Hague.)
Rullmann, H. (2003). Additive particles and polarity. Journal of Semantics, 20, 329–401.
Sadock, J. (1972). Speech act idioms. In *CLS 8*, 329–339.
Searle, J. (1975). Indirect speech acts. (In P. Cole & J. Morgan (eds.) Syntax and Semantics: Speech Acts (pp. 59–82) New York: Academic Press.)
Simons, M. (2001). On the conversational basis of some presuppositions. (In R. Hastings, B. Jackson & Z. Zvolensky (eds.), Proceedings of Semantics and Linguistic Theory XI, Ithaca, NY: CLC Publications.)
de Swart, H. (1996). Meaning and use of *not ... until*. Journal of Semantics, 13, 221–263.
van der Wouden, Ton. (1995). A problem with the semantics of neg-raising. Ms.
van der Wouden, Ton. (1997). Negative Contexts: Collocation, Polarity and Multiple Negation. Routledge

Zwarts, F. (1996). Facets of negation. (In J. van der Does and J. van Eijck (eds.), Quantifiers, logic and language (pp. 385–421) Stanford: CSLI Publications.)
Zwarts, F. (1998). Three types of polarity. (In F. Hamm & E. Hinrichs (eds.), Plural quantification (pp. 177-238) Dordrecht, Kluwer.)